Supersonic nozzle profiling for low-temperature plasma generators: theoretical predictions and experimental evidence

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The present paper deals with the modified method for the supersonic nozzle profiling with respect to non-monotonic dependence of adiabatic index on temperature, as well as the results of nozzle profile calculation, based on independently determined specific heat curve for molecular nitrogen N_2 and products of its thermal decomposition in the temperature range of $T = 260...100000$ K and atmospheric pressure. The final section shows the results of axial velocity measurements for a free plasma jet outflowing from the nozzle developed.

1. Introduction

Plasma torches are widely used in manufacturing processes of plasma heating (particularly in supersonic and hypersonic aerodynamical testing), alloys and metal processing (melting, cutting, coating etc.) and for industrial waste disposal. Current issues of plasma torch modelling are the adequate consideration of MHD-effects in arc currents, intense turbulence generation due to a sharp heat emission (a fluent electric arc zone with the heat density of $w \sim 10^{11}$ W/m^3). High temperatures often require both correct transport coefficients and radiation transport calculation in view of non-equilibrium plasma.

Combustion and non-transferred electrical arc heating is often used to expand the available temperature and Mach number ranges of the facilities aimed for aerodynamic testing of supersonic and hypersonic cruise vehicles (HCV) whereas chemical reaction of equilibrium dissociation and ionization take place producing high-enthalpy flow. However, with further acceleration the recombination reactions develop, leading to a sharp nonmonotonic behavior of adiabatic index function. Dissociation and ionization become also important while investigating the shock-compressed layer at the leading edge of HCV wing and the spark discharge propagation [1].

In order to modify classical one-dimensional nozzle profiling technique [2] and implement it for the case of variable adiabatic index, as well as for modeling by the industrial CFD programs, we have performed the self-sustained calculation of the heat capacity of nitrogen and its degradation products in the temperature range of 260 – 100 000 K and atmospheric pressure based on the equilibrium dissociation and ionization equations Fig. 1 shows specific heat capacity of nitrogen versus temperature at constant pressure $p = 1$ bar for different number of chemical reactions involved: 3 – two reactions, 4 – five reactions, 5 – six reactions and comparison with other researchers: 2- IVTANTHERMO. Curve 1 shows the results of [3, 4] for air.

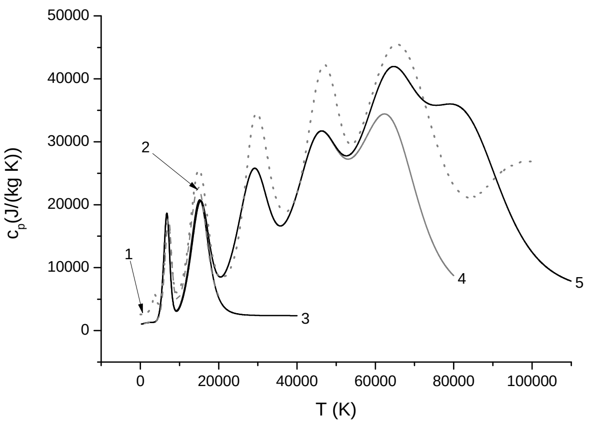

Fig. 1 Specific heat capacity of nitrogen versus temperature of various models

2. Supersonic nozzle profiling for low-temperature plasma generator

The flow in the nozzle regardless it's application is often assumed to have an axisymmetric structure that can be divided into three characteristic regions: the subsonic flow region in the tapered portion, transonic region in the vicinity of the critical cross section and the supersonic region in an expanding part of the nozzle. In the present study we deal with sufficiently small nozzle with the characteristic dimensions $L \approx 0.01$ m that urges to reject due to technological reasons the complex curved profiles, so the consideration is restricted by conical-wall nozzle.

Numerical simulation of the immersed plasma jets plays a substantial role in plasma physics and physical gas dynamics, as one need to account for the effects of the high-speed flows (shock waves), the significant number of molecular reactions, the material-radiation interaction and its impact in heat transfer along with of the electrodes and channel walls destruction and plasma turbulence.

Reynolds number at the plasma torch exit at the certain flow rates reaches $\mathrm{Re} \sim 10^3$, so the flow is turbulent and we have to take into account high turbulent mixing and eddy thermal conductivity, if the characteristic size of the cross section is $L \approx 10^{-2}$ m, then according to Kolmogorov microscale turbulence theory, the computational cell size should be about $l \approx 10^{-5}$ m. Evidently, the three-dimensional unsteady turbulent plasma jet calculation taking into account the radiation is a time consuming task, in terms of both algorithm complexity and the computational resources required.

The design of nozzles with known flow rate characteristics and thermodynamic parameters at the nozzle exit is an important issue in aerodynamic testing in supersonic and hypersonic flows. One can solve the inverse problem of the nozzle theory [5] using a well-developed numerical methods for nozzles with characteristic dimensions m. Among of them is widely used method of characteristics [6].

Conical nozzles are widely used in propulsion systems, wind tunnels, as well as in studies of various nonequilibrium processes. Profile conjugating characteristics of various nozzle sections can have a significant impact on the flow structure. For example [19] flow deceleration in the vicinity of radius-conical conjugation at the critical nozzle portion under certain conditions can give a rise to the stationary shock wave that is repeatedly reflected from the symmetry axis and the nozzle profile.

Elementary nozzle profiling technique in the case of constant adiabatic index is based on the given flow rate and stagnation parameters. The idea of method proposed is that for the case of variable adiabatic index one can use the law of enthalpy conservation and its temperature dependence calculated in advance.

Enthalpy is a thermodynamic system functional and can be reckoned from some arbitrary value. In particular case, it is defined as a function of the heat capacity from a certain temperature value $T_0 = 260$ K:

$$\mathrm{h(T)} = \int_{T_0}^{T} \mathrm{c}_p(\mathrm{T'})\,\mathrm{dT'}.$$

Using one of the elementary consequences of the equation of motion

$$\mathrm{h(T}_{stag}) = h(\mathrm{T}) + \frac{u^2}{2},$$

where T_{stag} is a stagnation temperature, one can profile conical-wall nozzle resting upon the inlet, critical and outlet cross-sections, given the mass flow rate G, static inlet pressure p_1, temperature T_1 and diameter D_1 in the settling chamber, as well as Mach number $M_{out} = 4$ of the outlet flow. At the first stage the calculation of missing parameters in the inlet section subscripted as «1» was performed (molar mass μ_1, sound speed a_1, Mach number M_1, enthalpy h_1), which were then used for stagnation enthalpy

$$h_{stag} = h_1 + \frac{1}{2}\left(\frac{GRT_1}{A_1 p_1}\right)^2,$$

where A_1 - nozzle cross section, and T_{stag} - stagnation temperature. Involving the adiabatic equation with an exponent index $k = k_{stag}$ one can determine other required thermodynamic variables.

Enthalpy of molecular nitrogen and its temperature dependence is the basis of suggested approach.

The proposed profiling method was used to calculate the supersonic nozzles for two sets of the thermodynamic parameters at the nozzle inlet.

Fig. 2 depicts the dependence of the supersonic nozzle diameter on Mach number (assuming the linear growth) at the inlet pressure: $p_1 = 4$ bar, temperature $T_1 = 10020$ K and gas flow rate $G = 0.016$ kg/s (marked in black). The black curve shows the results of 200 sections calculation, and the sign ★ marks a non-iterative implementation for the critical and outlet cross-sections, ▲ marks the calculation results based on [2]. Grey solid and dashed lines indicate the supersonic nozzle diameter dependency on the Mach number at a higher temperature. Gray circle shows the non-iterative results for the critical and outlet cross-sections, △ - results of calculations based on [2].

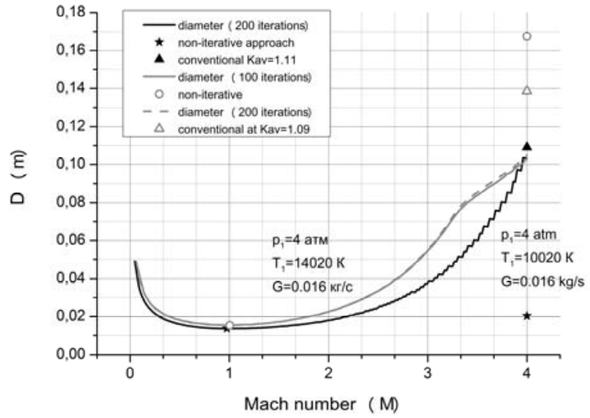

Fig. 2 Nozzle diameter as a function of Mach number

3. Supersonic nozzle velocity properties measurements

To verify the acceleration characteristics of supersonic nozzle profiled, as well as to determine the gas velocity, a series of measurements of stagnation pressure was conducted. As a source of high-enthalpy plasma flow we used a plasma torch with a rated power of 50 kW (subsonic channel diameter - 10 mm) with a swirl stabilization and expanding channel providing high flow rates, efficient heating of the working medium and small thermal losses in the water-cooled anode surface. The use of such low-temperature plasma generator allows to obtain plasma flow temperature in the range of thousands to tens of thousands of degrees that is of particular interest in plasma thermodynamics and chemistry.

As the heated gas accelerated up to the outlet of critical portion of the nozzle discharges to the atmosphere, it is exposed to the abrupt deceleration passing via shock wave that results in the subsonic outflow at high temperature. According to our spectroscopic measurements the corresponding temperatures are $T_1^{out} = 5000 \pm 250$ K at the current $I_1 = 150$ A (power $W_1 \approx 14.25$ kW) and $T_2^{out} \approx 5500 \pm 250$ K at $I_1 = 200$ A ($W_2 \approx 18$ kW). The flow rate in both cases was $G = 1$ g/s.

The stagnation pressure measurements were performed using a Pitot tube consisting of a curved capillary pipe, enclosed in a copper water-cooled body frame. The probe mounted on the three-degree-of-freedom traversing gear with a positioning accuracy of $\delta\overline{x} = \delta\overline{y} = \delta\overline{z} = 0.1$ mm. The stagnation pressure was measured by current Honeywell Eclipse OEM Pressure Transducer with an operating range $p = 0...20.413$ bar, base current of $i_0 = 4$ mA and a linear pressure-current characteristics $dp/dt = 1.2927 \times 10^8$ Pa/A. The current and temperature measurement errors are estimated as $\delta i = 1\ \mu A$ and $\delta T = 250$ K correspondingly.

As a preliminary procedure Pitot tube alignment along the jet axis and zero point adjustment was undertaken. After that the probe positioned 100 mm below the nozzle exit to measure the stagnation pressure sequentially with a spatial step $\Delta x = 10 \pm 1$ mm at two different current values $I_1 = 150$ A and $I_1 = 200$ A. Based on the current measurements obtained and the spectroscopic data on temperature, it is possible to determine the density $\rho = p / RT$ (p - static (atmospheric) pressure, T - temperature, $R(\mathrm{T})$ - the gas constant), and the velocity $u = \sqrt{2(\mathrm{p}_0 - \mathrm{p})/\rho}$, where $p_0 - p$ is pressure difference measured by the transducer. In addition, defining the speed of sound at the nozzle exit $a^{out} = \sqrt{k(\mathrm{T}^{out})\,\mathrm{R}(\mathrm{T}^{out})\,\mathrm{T}^{out}}$ equal to $a_1^{out} = 1291.3$ m/s in the first case and $a_2^{out} = 1359.6$ m/s in the second one can plot the reduced velocity versus the distance from the nozzle exit measured in calibers.

It is worth noting that due to low N_2^+ spectral lines intensity even at a small distance from nozzle exit we managed to obtain temperature as the average domain value in the vicinity of the nozzle exit. Thus, the respective densities are: $\rho_1^{out} = \rho(\mathrm{T}_1^{out}) = 0.0666$ kg/m^3, $\rho_2^{out} = 0.0576$ kg/m^3.

Two graphs were constructed both in dimensional (u, z) and dimensionless (λ, z'), $z' = z/D^{out}$ coordinates, where $D^{out} = 6$ mm - the outlet section diameter. The corresponding fitted curves were determined by the least square method.

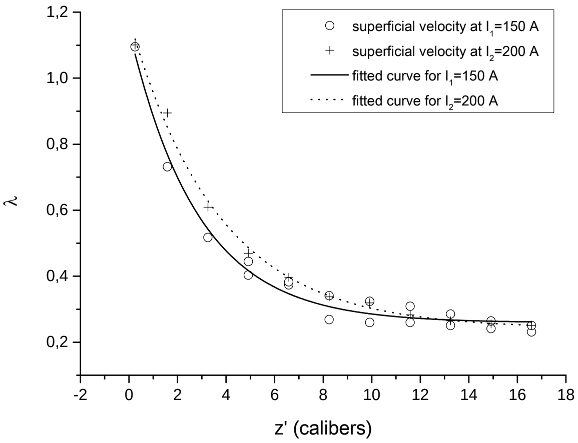

Fig. 3 Axis reduced velocity versus the dimensionless distance from nozzle exit

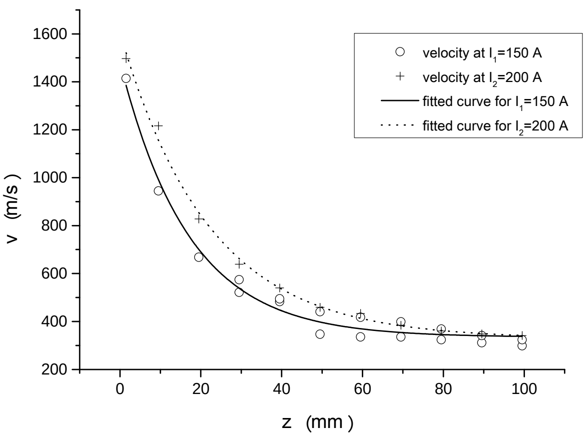

Fig. 4 Axis velocity versus the distance from nozzle exit

The experimental data are easily approximated by a decreasing exponential function. As expected, increasing heat emission (or electric arc power) leads to the rise of gas flow velocity at the central axis of the nozzle. Turning to Fig. 7 one can see that at the nozzle exit the transonic regime is implemented, as in this domain the reduced velocity equals to the Mach number M. We also assume that at large distances from the nozzle exit transonic regime still persists because of the jet cooling leading to the sound speed reduction. This speculation has not yet been verified experimentally in view of the low radiation intensity of the molecular ion N_2^+, which may require to come over to other spectral ranges.

The differences in the arrangement of the curves at various power consumptions almost disappear at a distance of 10 calibers, which is probably caused by the temperature drop and mixing of the jet with the ambient air.

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