Resonance radiation transport in the contracted discharge in argon

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Self-consistent numerical modeling of a positive column of the argon glow discharge at tens of Torrs is performed. At these conditions, the discharge appears in diffuse and contracted modes. To obtain the hysteresis of the discharge parameters and unstable branches, a new approach is used, which gives the dependence of the discharge current on the electric field (an inverse dependence). Propagation of resonance radiation (radiation to a ground state) inside the discharge influences the distribution of resonance atoms due to effective reabsorption (this effect is called "Resonance radiation transport"). The distribution of resonance atoms is calculated by solving the Biberman-Holstein equation. It is shown that the traditional "effective lifetime" approximation for the balance of resonance atoms gives significant errors, in particular, in the effective value of a filament area.

1. Introduction

The contraction of a positive column, in particular, in inert gases, is an important problem in gas discharge physics. There are different models of contraction, including inhomogeneous gas heating and the nonlinear dependence of the ionization rate on the electron density due to electron-electron collisions. Theoretical and experimental results related to the contraction phenomenon in inert gases have been reviewed in [1].

The particle transport mechanisms are important in the description of the contraction because they determine the filament radius. However, transport of resonance atoms due to reabsorption of resonance radiation is usually neglected: these atoms are described by "effective lifetime" approximation.

In this work, the contraction is explained by nonlinear dependence of the excitation rate and other rate constants on the electron density due to electron-electron collisions. Resonance atoms are described by Biberman-Holstein equation, and the effect of radiation transport is discussed.

2. Kinetic scheme

In argon discharge, at pressures higher than 10 Torr, the reduced electric field does not exceed 1 V/(cm·Torr) and the electron distribution function (EDF) is formed locally, which allows us to apply a fluid model for the description of the column.

At not very high currents, only four lower states (two metastable and two resonance states) play a significant role in the ionization balance. These states are mixed by electrons [2], and the radial distribution of excited atoms is determined by states with a lowest lifetime, that is, by resonance states.

Inhomogeneous gas heating is not a primary reason of the discharge contraction [1]. In the present work, gas heating is not taken into account.

The present model includes the ground state of Ar, two metastable (s_5, s_3) and two resonance (s_4, s_2) states, the molecular ion Ar_2^+, and the electron.

The processes included in the model are listed in Table 1. The acronym BE means "Calculated by Boltzmann equation".

Table 1. Elementary processes

Notation	Rate constant
$Ar + e \to Ar + e$	BE
$Ar + e \to Ar^* + e$	BE
$Ar + e \to Ar^+ + 2e$ ($\to Ar_2^+$)	BE
$Ar^* + Ar^* \to Ar_2^+ + e$	$6.4\cdot10^{-10}$
$Ar_2^+ + e \to Ar^* + Ar$	$9.1\cdot10^{-7}\,(300/T_e)^{0.61}$
$Ar(s_4) \to Ar + h\nu$	$1.2\cdot10^{8}$ s^{-1}
$Ar(s_2) \to Ar + h\nu$	$5.1\cdot10^{8}$ s^{-1}
$Ar^* \to$ wall	40 cm^2/(s·Torr)

3. Equations

The electrons are described by an ambipolar diffusion equation:

$$-\frac{1}{r}\frac{d}{dr}\left(rb_i\frac{kT_e}{e}\frac{dn_e}{dr}\right)=n_e\sum_k N_k\nu_k^{step}-\alpha n_e^2 \quad (1)$$

Here b_i is the ion mobility and T_e is the electron temperature. The electrons are produced in stepwise ionization processes (its rate constant is ν_k^{step}) and destroyed due to the ambipolar diffusion and the bulk recombination (its rate constant is α). The boundary condition is zero flux at r=0 and zero density at r=R.

Metastable atoms play an important role in the ionization balance. They are described by a diffusion equation including excitation and destruction:

$$-\frac{1}{r}\frac{d}{dr}\left(rD_k\frac{dN_k}{dr}\right)=n_eW_k-$$
$$n_eN_k\left(\nu_k^{step}+\sum_i\nu_{ki}\right)+n_e\sum_i N_i\nu_{ik} \qquad (2)$$

Here D_k is the diffusion coefficient, W_k is the excitation rate by electrons, and ν_{ik} is the frequency of transition from *i*-th to *k*-th state by electron impact.

Resonance states are described by Biberman-Holstein equation [1]:

$$A_kN_k-A_k\int\limits_{(V)}N_k(r')K(r',r)d^3r=N_eW_k-$$
$$n_eN_k\left(\nu_k^{step}+\sum_i\nu_{ki}\right)+n_e\sum_i N_i\nu_{ik} \qquad (3)$$

In this equation, A_k is the transition probability and K is the kernel of the integral operator describing the radiation transport.

Eq. (3) can be reduced to the system of linear equations even in a case of high optical density [3].

4. Method of solution

The system of differential and integral equations (1-3) can be reduced to a system of linear equations by using the method of finite differences and the "matrix" method of solution of Biberman-Holstein equation described in [3].

This system of equations includes rate constants of excitation and ionization by electrons calculated by the solution of Boltzmann kinetic equation. In presence of elastic, inelastic and electron-electron collisions, the Boltzmann equation is nonlinear and can be solved by the iterative method. The EDF depends on two parameters E/N and n_e/N (N is the ground state density) and the rate constants can be stored as tables.

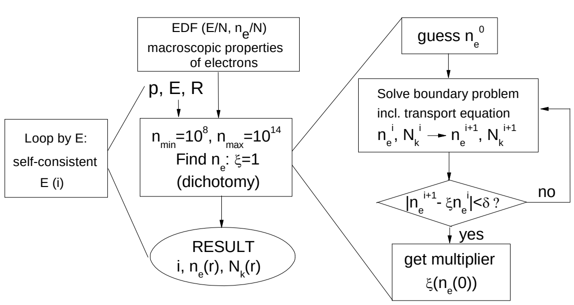

Figure 1. Scheme of the self-consistent calculation of the electron and excited atom densities.

The set of equations (1-3) is also nonlinear and requires some iterative method. Note that all variables included in this set of equations depend on electric field which is constant along the radius.

In the present work, the following method is used to solve the system of equations (Fig. 1). At fixed electric field E, the system (1-3) is linearized relative to n_e and N_k, and next iteration is calculated by the solution of a linearized system. It has been found that, after some iterations, the radial profile of densities converges but the absolute value of the density differs by a factor ξ. This factor depends on E; at a certain value of E this factor is equal to 1, this is a self-consistent electric field.

This method allows us to obtain self-consistent electron density (and therefore the current) for each value of E. By inversing this function, we can get a self-consistent volt-ampere characteristic of the discharge. Since the discharge has two modes (diffuse and contracted), volt-ampere characteristic may include hysteresis and unstable branches; the advantage of our method is that, besides stable branches, the unstable ones can also be found.

5. Results and discussion

The contraction in argon appears at p=10 Torr and higher, i=10 mA and higher. The present work is related to pressure 30 Torr and i = 0.1 – 50 mA.

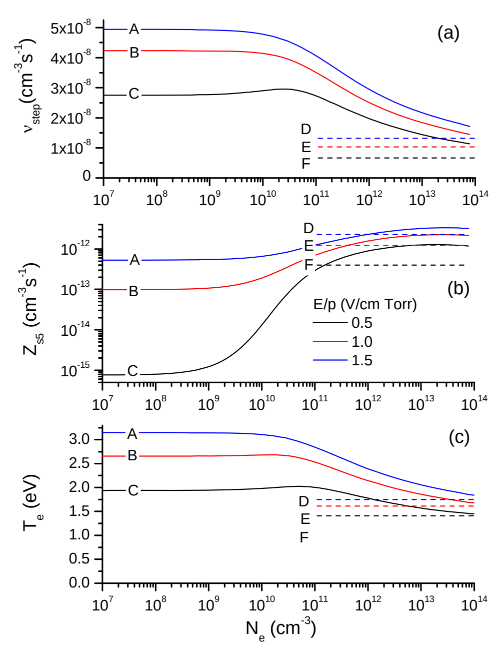

Figure 2. Stepwise ionization (a), excitation of s_5 state (b), and electron temperature (c) vs. n_e, for E/p=0.5 (A, D), 1.0 (B, E), 1.5 (C, F) V/(cm Torr).

In this range of currents, electron-electron collisions cause the Maxwellization of the EDF (the number of fast electrons increases), and the excitation rate nonlinearly grows with electron density (Fig. 2(b)). Such values as T_e and ν_{step}, which are determined by low-energy electrons, slightly decrease with electron density. However, rapid growth of excitation with density is enough to cause the discharge contraction.

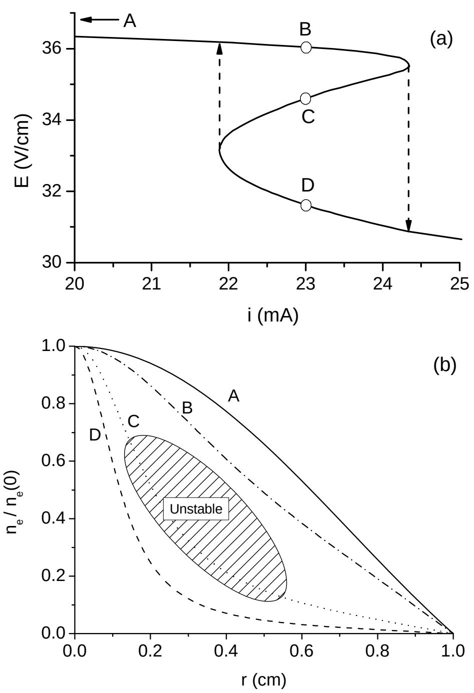

Figure 3. (a) Dependence of the electric field on current, points denote different discharge states. (b) Radial dependences of electron density on radius.

The transition of a diffuse discharge mode into a contracted one is shown in Fig. 3. The discharge is diffuse and the profile of electron density is close to Bessel profile up to i=20 mA (curve A). At i=23 mA, the electron density profile shrinks due to bulk recombination (curve B) and finally the discharge abruptly turns into a constricted state (curve D).

Since the electron density in a contracted discharge exceeds its value in a diffuse mode by two orders of magnitude, the electric field required to maintain the discharge abruptly decreases (see dashed arrow).

A hysteresis is observed in the volt-ampere characteristic of the discharge, which means, the backward transition of a contracted discharge to a diffuse mode occurs at a lower current. Point C (partially contracted discharge) belongs to an unstable branch and can't be observed. However, the present model allows us to obtain even an unstable branch of the volt-ampere characteristic.

The effect of the radiation transport can be studied by comparing the results of two models: accurate modelling and "effective lifetime" approximation. The latter does not take into account any radiation transport, assuming the lifetime of s4 and s2 atoms to be

$$A_{eff} = C \frac{A}{\sqrt{\pi k_0 R}} \tag{4}$$

Here k_0 is the absorption coefficient in the center of a spectral line (the line profile is assumed to be Lorentzian), and R is the tube radius. The constant C≈1 has slightly different values in works of Biberman and Holstein.

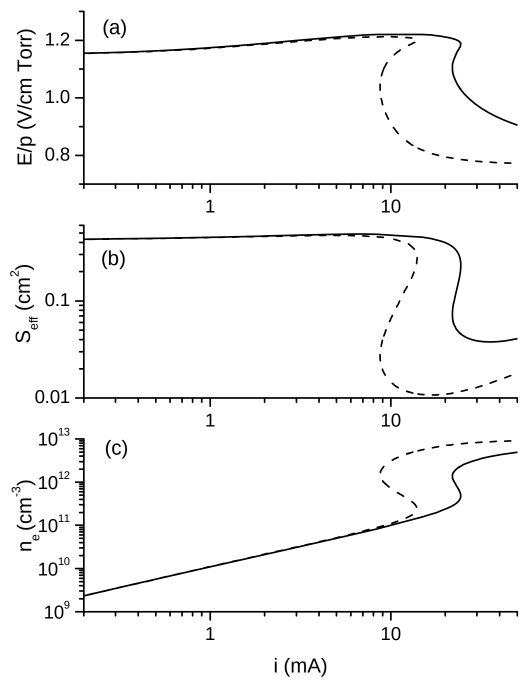

Figure 4. Reduced electric field (a), effective area of the filament (b) and electron density at r=0 (c) at p=30 Torr with (solid) and without radiation transport.

The results of the comparison are represented in Fig. 4. It is seen from this figure that radiation transport slightly stabilizes the discharge (the transition to a contracted state occurs at higher currents). The radiation transport causes widening of the filament area in a contracted discharge, it is 3 times higher than the result of an effective lifetime approximation.

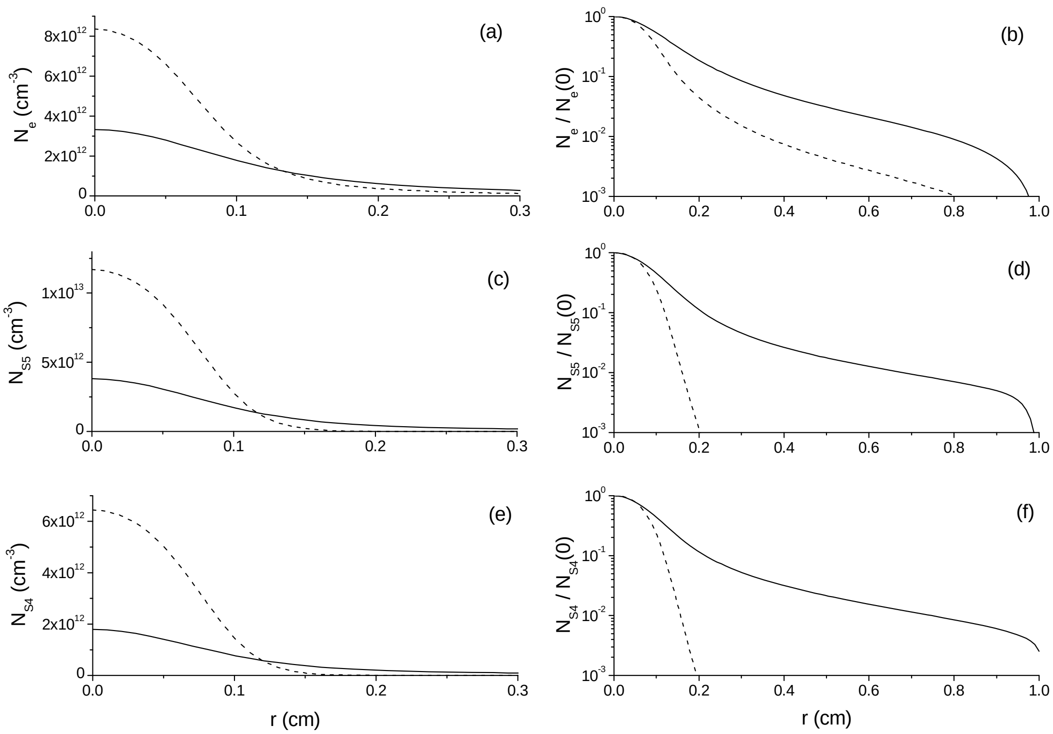

Figure 5. Radial dependences of the electron density (a), (b), density of the metastable state s5 (c), (d) and density of the resonance state s4 (e), (f) in a contracted discharge (p=30 Torr, i=30 mA). Left charts, linear scale; right charts, logarithmic scale (normalized to unity at r=0). Solid line corresponds to radiation transport model an dashed one corresponds to an "effective lifetime" approximation.

The differences in the effective lifetime approach and accurate solution of the Biberman-Holstein equation can be seen in Fig. 5. Radiation transport causes essential widening of the excited atoms and therefore the widening of a filament.

It is seen in Fig. 5 (d) and (f) that radiation transport is a nonlocal process, because excited atom densities are nonzero in a whole discharge volume.

Note that excited atom densities in a contracted mode are proportional due to the intensive mixing by electrons. In the presence of mixing, radial profiles are determined by states with a smallest lifetime, i.e., by resonance states. Thus, accurate consideration of their transport is very important.

6. Conclusion

The transition from a diffuse to a contracted discharge in argon is demonstrated by modelling in a range of currents 0.1 – 50 mA at p = 30 Torr. The primary reason of the contraction is a nonlinear dependence of excitation rate on electron density and a bulk recombination. The dependences of the discharge parameters on the current show a hysteresis due to abrupt contraction.

The influence of resonance radiation transport on the properties of a contracted discharge is demonstrated, and the importance of accurate solution of the Biberman-Holstein equation for proper description of the contraction phenomenon is shown.

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8. References

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