Striations in noble gas discharges as spatial resonator modes

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A new approach to describe striations in noble gas discharges is proposed. From this viewpoint, the positive column is a spatial resonator, in which spatial periods of electric fields are considered as mode lengths: fundamental (L_S) or higher ones ($\frac{q}{p} L_S$, where *q,p* are integers and $p > q$). These distances are equivalent to lengths of striations of different types. Resonance trajectories of electrons in the "coordinate-energy" phase plane are depicted using a point mapping method. Mechanism of stratification can be explained the following way: an arbitrary EEDF is injected in a column from a cathode layer, and then response of the column considered as resonator is interpreted as appearance of one of the resonator modes. Appearance of the certain striation type depends on discharge conditions. Comparison of discrete model results with kinetic theory ones is performed. The new approach provides more simple solutions than the kinetic theory for the energy relaxation lengths which can even exceed the positive column length.

2. Positive column as a spatial resonator

Stratification of the glow discharge is a fundamental problem in the field of non-equilibrium plasmas. There are two main approaches to describing stratification nowadays – a fluid and a kinetic one. In the current work striations considered using the nonlinear dynamics approach.

An electron motion in the constant field of positive column can be depicted the following way. Electrons, being accelerated in the field E_0, gain energy equivalent to the excitation threshold ε_{ex} across the path length L_0. Then electrons undergo inelastic collisions and lose an energy quantum equivalent to ε_{ex}, then accelerate again and this process repeats periodically. Hence, one can introduce a period of energy loss in inelastic collisions $L_0 = \varepsilon_{ex} /(eE_0)$. The role of small energy losses in elastic collisions will be considered below. One consider electron trajectory in a sine-modulated field

$$E(z) = E_0\left(1 - \alpha\cos\frac{2\pi z}{L}\right) \quad (1)$$

where *L* is an arbitrary period length, α is the field modulation depth, and potential energy has the form

$$V(z) = -eE_0 z + \alpha\frac{eE_0 L}{2\pi}\left(\sin\frac{2\pi z}{L}\right) \quad (2)$$

An electron in the phase plane "coordinate – full energy" (z,ε) after *n* motion cascades gains energy

$$\varepsilon_n = w_0 - n\varepsilon_{ex} = \varepsilon_{ex} + V(z_n) \quad (3)$$

where w_0 is an initial kinetic energy. At the same time, energy at the point $z_n - 0$ is

$$\varepsilon_n = w_0 - n\varepsilon_{ex} = \varepsilon_{ex} + V(z_n),$$

and in the point $z_n + 0$ is $\varepsilon_{n+1} = \varepsilon_n - \varepsilon_{ex}$.

Substituting the expression for potential (2) into (3), one obtains an equation for the determination of the points $z_n(L, w_0)$

$$w_0 - (n+1)\varepsilon_{ex} + eE_0 z_n - \alpha\frac{eE_0 L}{2\pi}\left(\sin\frac{2\pi z_n}{L}\right) = 0 \quad (4)$$

$n = 0,1,2...$

The phase angle is defined as $\vartheta_n = \dfrac{2\pi z_n}{L}$. Considering this fact in (4), equation takes the form

$$\alpha\cdot\sin(\vartheta_n) = \vartheta_n + \theta - 2\pi(n+1)\frac{L_0}{L}, n = 0,1,2... \quad (5)$$

Here $\theta = \dfrac{2\pi w_0}{eE_0 L}$ correspond to the initial phase angle.

Analysis of the equation (5) shows that rational values of L_0 / L provide periodical solutions, and irrational values provide non-periodical solutions. Differences between these cases are demonstrated in the Fig.1 (specifying $\alpha = 1$).

One can see that in the case of $L = {}^5\!/_6 L_0$ solution is periodical and in the case of $L = {}^{\sqrt{24}}\!/_6 L_0$ points are located non-periodically, despite of close ratio values.

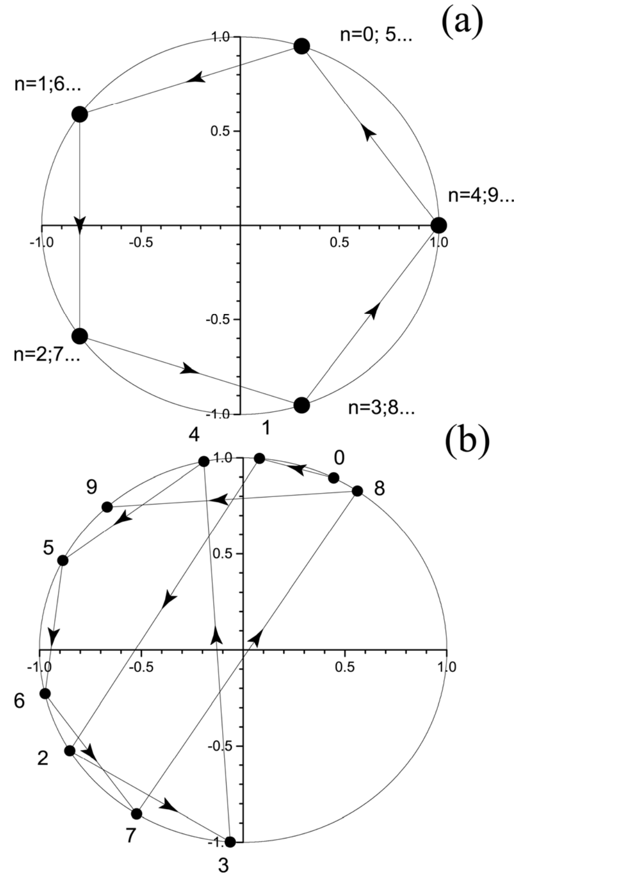

Fig.1. Change of the phase angle ϑ **(**in radians) during the period of field modulation for the cases $L = \frac{5}{6}L_0$ (*a*) and $L = \frac{\sqrt{24}}{6}L_0$ (*b*). Phase angle $\vartheta_n, n = 0,1,2,\ldots$ is marked as filled dots at unit circle.

According to the described above, a generation possibility of striations with lengths which are rationally related with L_0 does not depend on discharge column length.

3. Resonance trajectories and resonances

One can show how the initial EEDF changes. For this purpose a point map (Lamerey diagram) can be constructed. Replacing θ by ϑ_{n-1}, L_0 / L by p / q, and introducing the new variable $Y_n = f(\vartheta_n) = \frac{2\pi np}{q} - \vartheta_n$, the equation (5) transforms into expression for the point map

$$(-1)^{2pn/q} \sin Y_n = Y_n - Y_{n-1}, \quad n = 1,2,3...$$

Using this formula the point maps for different resonances: integer S-resonance (*p*=1, *q*=1), integer P-resonance (*p=2*, *q=1*), and non-integer R-resonance (*p=3*, *q=2*), can be obtained (Fig.2*a-c*). Each of these resonances correspond to a real striation type (S-,P- and R-type respectively).

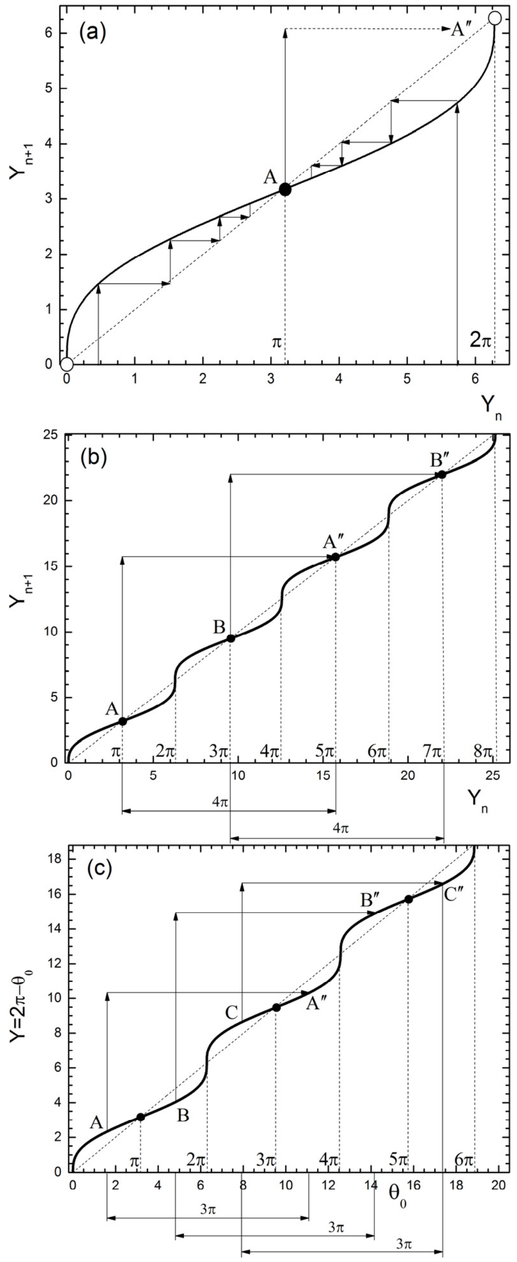

Fig.2. Point maps (Lamerey diagrams) for (*a*) S-resonance, (*b*) P-resonance and (*c*) R-resonance. Stable fixed points (in which inelastic collisions take place) are marked by filled circles, unstable fixed points – by open circles. The pairs of points A, A''; B, B'' and C, C'' belong to different resonance trajectories.

The trajectories passing $A \to A'' \to \ldots$ and $C \to C'' \to \ldots$ are qualitatively the same so they can be interpreted as one which appears two times more often than trajectory passing $B \to B'' \to \ldots$ It leads to an assumption that EEDF maxima corresponding to frequently appearing trajectories will be proportionally higher than infrequent (two times higher in the cases described above).

All described resonances correspond to the real striations in noble gas discharges.

4. Comparison with kinetic theory

An analytical theory of discharge stratification was proposed by L.D. Tsendin [1,2]. In this theory a Boltzmann kinetic equation is considered in the two-term approximation with so-called "black wall" condition at the excitation threshold. In this case kinetic equation is written as

$$\frac{\partial}{\partial z}D(w)\frac{\partial f_0(\varepsilon,z)}{\partial z}+\frac{\partial}{\partial \varepsilon}G(w)f_0(\varepsilon,z)=0, \qquad (7)$$
$$f_0(\varepsilon,z=0)=f_0^0(w), \quad f_0(\varepsilon,z)_{|w=\varepsilon_{ex}}=0.$$

Here f_0 is an isotropic part of the EEDF, $D(w)$ is a diffusion coefficient and $G(w)$ is a coefficient of energy loss in elastic collisions.

Using the parameter $\kappa=\frac{\varepsilon_{ex}}{\sqrt{\frac{M}{m}}eE\lambda}<<1$ equation (7) is solved by series expansion. The first term of this expansion corresponds to the equation solution neglecting the energy loss in elastic collisions ($\kappa=0$) and has the form

$$f_0(\varepsilon,z)=\Phi(\varepsilon)\int_z^{z_{ex}(\varepsilon)}\frac{dz}{D(w(z))}$$

Here $\Phi(\varepsilon)$ is a shape function. The energy loss ε_{ex} due to inelastic collision can be described by the equation

$$\Phi(\varepsilon-\varepsilon_{ex})=\Phi(\varepsilon) \qquad (8)$$

After taking elastic collisions into consideration and performing some transformations the differential equation is obtained:

$$\Phi(\varepsilon-(\varepsilon_{ex}+\Delta\varepsilon))=\Phi(\varepsilon)+$$
$$+\frac{\partial}{\partial\varepsilon}\left(\kappa\cdot\Phi(\varepsilon)\Psi(\varepsilon)+\kappa^2\frac{\partial}{\partial\varepsilon}\Phi(\varepsilon)C(\varepsilon)\right) \qquad (9)$$

There are an integral dependence of functions $\Psi(\varepsilon)$ and $C(\varepsilon)$ on $D(w)$ and $G(w)$.

On the basis of the equation (9) the presence of integer resonances, energy relaxation of electrons and "bunching effect" were described. Tsendin admitted that the trivial equation (8) can describe an establishing periodicity of an arbitrary initial EEDF. But in subsequent works only the equation (9) was considered, and trivial equation (8) has not been studied. However, all positive column resonance properties can be described using only the simple equation (8).

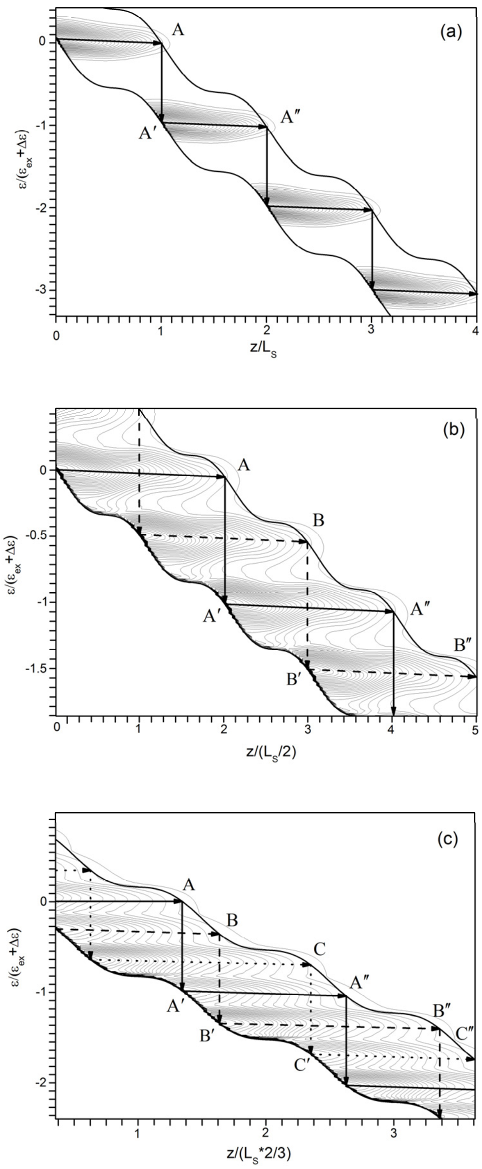

Fig.3. Illustration of the EEDF evolution and resonance trajectories for S- (a) P- (b) and R-striations (c) obtained from kinetic theory calculations.

Taking the initial EEDF in the form of δ-like function $f_0^0(w)=A\delta(w-w_0)$, considering $\Phi(\varepsilon)$ as the initial EEDF $f_0^0(w)$ and assuming $\kappa=0$, the solution of equation (7) is

$$f_0(\varepsilon,z)=A\delta(\varepsilon-n\varepsilon_{ex}-w_0)\int_z^{z_{ex}(\varepsilon)}\frac{dz}{D(w(z))} \qquad (10)$$

This expression describes the transition of the δ-like distribution along the phase trajectory. Such distribution has an amplitude maximum at $w=0$ and becomes zero at $w=\varepsilon_{ex}$.

So, the simple equation (8) describes all resonance properties of the positive column. The established periodical EEDF can be represented as a Gauss-like function $\Phi(\varepsilon)$ by the recurrent relation $\Phi(\varepsilon-(\varepsilon_{ex}+\Delta\varepsilon))=\Phi(\varepsilon)$ which is independent on the initial distribution $f_0^0(w)_{|z=0}$ and equation (9) transforms into (8). This equation describes the resonances with the fundamental period length $L_s=(\varepsilon_{ex}+\Delta\varepsilon)/eE$.

Also, there were works with the numerical calculations of the EEDF evolution in sine-modulated fields with high amplitude [3]. The non-integer resonances were found in these works. The evolution of the EEDF in periodical resonance fields can be described from the viewpoint of the kinetic theory by the following way. At first, one solved the kinetic equation for the isotropic part of the EEDF $f_0(\varepsilon,z)$ in the form

$$\begin{aligned}&\frac{\partial}{\partial z}D(w)\frac{\partial f_0(\varepsilon,z)}{\partial z}+\frac{\partial}{\partial\varepsilon}G(w)f_0(\varepsilon,z)=\\&=\sum_k wNQ_k(w)f_0(\varepsilon,z)-\\&-\sum_k(w+\varepsilon_k)NQ_k(w+\varepsilon_k)f_0(\varepsilon+\varepsilon_k,z)\end{aligned} \quad (11)$$

where *N* is the gas atom density and $Q_k(w)$ is the *k-th* level excitation cross section.

At second, one performed analysis of the electron relaxation. At this step a formation of resonance trajectories and convergence of an initial EEDF to narrow maxima moving along these trajectories can be seen and described.

Experimental study of the EEDF in striations and comparison with calculations were performed [4]. These measurements proved the convergence of the initial EEDF to narrow maxima moving along the resonance trajectories for all experimentally known striation types.

The nonlinear dynamics provides more simple approach, in which all resonance properties can be considered using the simple equation (5). The behavior of electrons in inhomogeneous fields, which is consisted of energy gain to the excitation threshold and abrupt energy loss due to inelastic collusion, is the same for both approaches (kinetic and discrete). So, the correlation between results is expected and is confirmed comparing of Fig.2 with Fig.3. Resonance trajectories in point maps are the same with trajectories in the kinetic theory calculations.

5. Conclusion

The presented approach, based on analysis of the phase trajectories, provides simple adaptation of the discharge stratification problem and also allows to avoid difficulties which appear in the kinetic theory at the large electron relaxation lengths.

In the kinetic theory, the length of relaxation of an arbitrary initial EEDF to the periodic one is long (even can be longer than the length of the positive column) and depends on the gas density. In the discrete approach the positive column is a spatial resonator, and striations of different types are considered as resonator modes.

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