Runaway electrons behaviour

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Runaway electron behaviour have been investigated both analytically and numerically. The electron energy growth equation was derived, and critical energies calculated analytically. In numerical simulations, the initial Maxwell distribution was deformed in a typical manner. The runaway electrons were accelerated in the presence of longitudinal magnetic field (parallel to the electric one), which in non-relativistic case does not influence the motion of the electrons along the electric field lines. Our simulations show, that this is not true in the relativistic case.

1. General

Runaway electron population originates from the Maxwell distribution tail under condition that electric field acceleration overcomes the collision processes. Such electrons can be accelerated to the considerable energies and are harmful for various experimental devices, e.g. tokamaks [1]. Runaway electrons are observed not only in laboratory plasmas, but also for example during lightning phenomena, in van Allen Belts, during plasma jet interaction with plasma background, etc.

Under specific conditions, ions or protons can be brought to the runaway regime as well. During storms such protons penetrate into atom nuclei and change it to radioactive isotope. Such mechanism is thought to be responsible for X-ray emission during thunderstorms [2].

The energy of the runaway electrons can be radiated as synchrotron radiation (in presence of magnetic field), used for avalanche mechanism (accelerated electrons boost normal electrons to the runaway regime), wasted for creation of electron-positron pairs [3] or deposited on the wall of the plasma device, e.g. in tokamaks the interaction of runaways with the wall leads to X-ray bursts.

The runaway electrons are detected very often, but their connection with various types of instabilities is not fully understood as well as their behaviour in strong magnetic fields and ultra relativistic conditions (energy much higher than the rest one).

2. Theory behind

The electron motion can be described by simple equation with electric and collision terms on the right hand side

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_\mathrm{e}v\right) = eE - \frac{Z^2 e^4 n_\mathrm{e}}{4\pi\varepsilon_0^2 kT}\ln\Lambda\, G\left(v/v_0\right), \qquad (1)$$

where $G(x)$ is so called Chandrasekhar friction function given by the well-known formula

$$G(y) = \frac{2}{\sqrt{\pi}}\frac{\int_0^y \mathrm{e}^{-x^2} x^2\,\mathrm{d}x}{y^2}, \qquad (2)$$

v_0 is thermal velocity $(2kT/m_\mathrm{e})^{1/2}$, and other symbols have usual meaning. The equation of motion (1) can be derived from Fokker-Planck equation and was analyzed in detail by Dreicer in 1959 [4].

If the electric field is higher than Dreicer field E_D, electrons are accelerated without let up. In smaller fields three regimes occur, see Fig. 1. From regime I (II) electrons are accelerated (decelerated) to the velocity v_1, which represents Ohmic regime. Particles with velocity higher than v_2 belong to the runaway population.

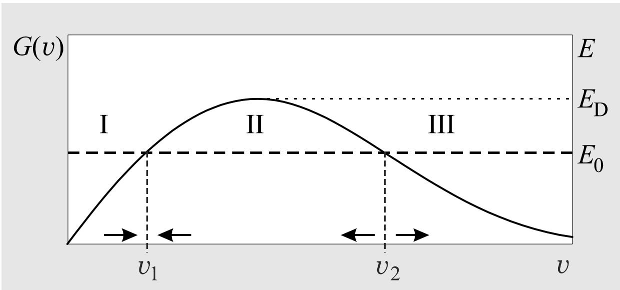

Fig. 1. Chandrasekhar function and electron acceleration regimes.

According to [5], the relativistic phenomena are important for fields fulfilling condition

$$\frac{E}{E_\mathrm{D}} \approx \frac{kT}{m_\mathrm{e}c^2}. \qquad (3)$$

In such a case the Chandrasekhar function tail changes its shape, and a secondary small maximum (bump) arises as a consequence of the synchrotron radiation onset.

According to [6], the influence of magnetic fields can be included as a small correction in the Coulomb logarithm Λ only, where Larmor radius is used instead of the Debye length.

Very fast electrons originate in presence of different instabilities, e.g. generalized Buneman instability [7], [8], during sudden tokamak disruptions [1], and fan/Parail-Pogutse instability.

3. Non relativistic energy growth equation

Let us replace the Chandrasekhar function by

$$g(y) = \frac{2y}{3\pi^{1/2} + 4y^3}\,. \tag{4}$$

Function $g(y)$ has the same limit behaviour in the vicinity of both zero and infinity, and its course is very similar to the Chandrasekhar one.

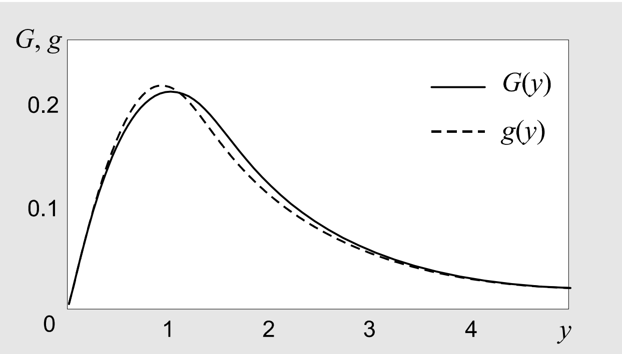

Fig. 2: The simpler form of the Chandrasekhar function.

Equation of the motion can be rewritten (using dimensionless variables) into the form

$$2\frac{\mathrm{d}\bar{v}}{\mathrm{d}\bar{t}} = \bar{E} - \frac{\bar{v}}{1 + (4/3\pi^{1/2})\bar{v}^{\,3}}\,, \tag{5}$$

where the normalization is defined as

$$\bar{v} = \frac{v}{v_0};\quad \bar{E} = \frac{e}{m_\mathrm{e} v_0 \nu} E;\quad \bar{t} = 2\nu t;\quad \bar{x} = \frac{2\nu}{v_0} x. \tag{6}$$

We denote the constant part of the electron-ion collision frequency as

$$\nu = \frac{Z^2 e^4 n_\mathrm{e}}{6\pi^{3/2} 2^{1/2} \varepsilon_0^2 m_e^{1/2} (kT)^{3/2}} \ln\Lambda\,. \tag{7}$$

The equation (5) can be simply transformed to the normalized energy equation by the substitution

$$\bar{w} = \bar{v}^{\,2}\,. \tag{8}$$

Furthermore we will use the distance as the independent variable. Thus, result is

$$\frac{\mathrm{d}\bar{w}}{\mathrm{d}\bar{x}} = \bar{E} - F(\bar{w})\,;\,, \tag{9}$$

where

$$F(\bar{w}) = \frac{\bar{w}^{1/2}}{1 + (4/3)\pi^{-1/2}\bar{w}^{3/2}}\,. \tag{10}$$

Function $F(w)$ is in its course alike with the Chandrasekhar function. For the energy dependence on the distance we need to solve the equation

$$F(\bar{w}) = \bar{E}\,. \tag{11}$$

Under the condition

$$\bar{E} < \max_{<0,\infty)} F = \frac{\pi^{1/6}}{9^{1/3}}\,, \tag{12}$$

the cubic equation (10) has two real solutions:

$$\bar{w}_{1,2} = \frac{\pi^{1/2}}{\bar{E}} \cos^2\left(\frac{\pi \pm \varphi}{3}\right), \tag{13}$$

where

$$\varphi = \arccos\left(3\pi^{-1/4}\bar{E}^{3/2}\right). \tag{14}$$

Energies w_1, w_2, has similar meaning as velocities v_1, v_2 in Fig. 1. Above the w_2 value the electron energy increases regardless of the collisions. If

$$\bar{E} \geq \max_{<0,\infty)} F\,, \tag{15}$$

the system is in unconditional runaway regime.

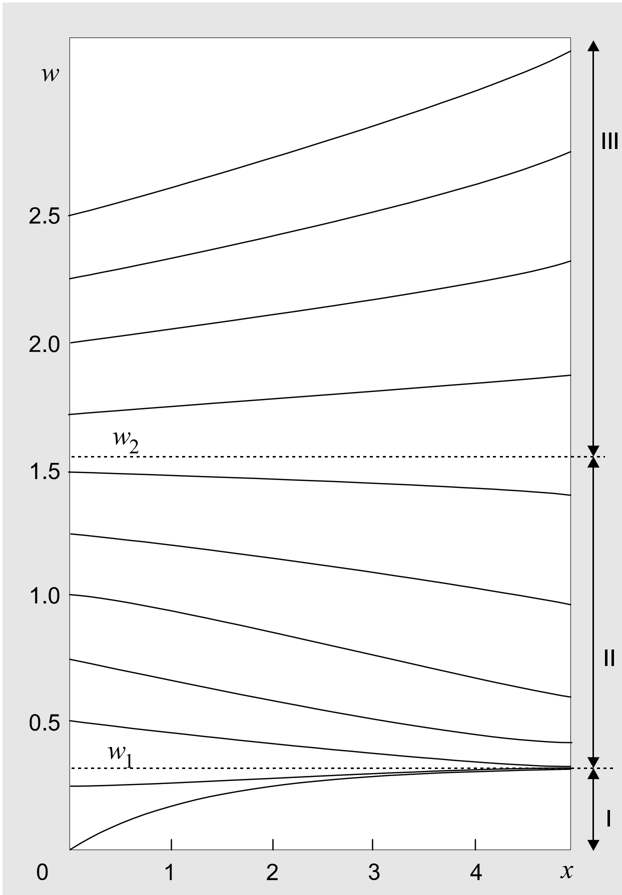

Fig 3. Numerical solutions of the equation (9) for various initial conditions and $\bar{E} = 0.5$.

The new electron energy equation (9) derived above can be used for calculations of the electron energy growth until the electrons reach the fully relativistic regime. The quantity w represents the kinetic energy of electrons in units of kT.

4. Relativistic numerical simulations

In relativistic case the Chandrasekhar function is no more defined by formula (2) and the equation of motion must be rewritten into relativistic form. Magnetic field also influences the particle motion:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma m_{\mathrm{e}}\mathbf{v}\right) = e\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) - S(\mathbf{v})\frac{\mathbf{v}}{v}, \qquad (16)$$

where

$$\gamma = \frac{1}{\sqrt{1-\mathbf{v}^2/c^2}}. \qquad (17)$$

is the relativistic factor and S is the collision term replacing Chandrasekhar function [10]. Numerical integration is simplified by substitution

$$\mathbf{u} = \gamma\mathbf{v} = \frac{\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}}. \qquad (18)$$

For simulations we used several difference schemes (Runge-Kutta, Boris-Buneman, Leap-Frog), all in relativistic versions. The mixing of velocities in the relativistic factor caused growth of the Larmor radius, even in the case of parallel electric and magnetic fields. It does not seem to be a numerical problem, as it occurred in all numerical solvers, and independently of the time step used.

5. Conclusion

A simple energy equation suitable for non-relativistic runaway electron energy calculations was found and critical energies derived. In relativistic case there were done numerical simulations which lead to the Larmor radius growth in ultra relativistic regime. It is open question whether this phenomenon really exists or it is compensated by synchrotron radiation. Validity of Lorentz equation in ultra relativistic regime must be tested as well as its possible replacement by Lorentz-Dirac equation with self-radiation term. These issues are subject of our current studies.

Acknowledgment

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