Oscillating solutions of the magnetized plasma-wall transition problem

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A one-dimensional fluid model of the magnetized plasma-wall transition is presented. The model is based on the continuity and momentum exchange equation for the ions, while the Boltzmann relation is assumed for the electrons. It is assumed that the ion flux in the whole system is constant. The solutions of the model are very sensitive to the boundary conditions and to the collision frequency of the ions.

1. Introduction

In recent years many papers have been published describing the magnetized plasma-wall transition region in front of a negatively biased wall immersed in plasma under the action of a constant magnetic field. Among these works is Chodura's model, which describes the pre-sheath of a collision-less plasma in an oblique magnetic field [1]. In his model, Chodura shows that there exists a magnetic pre-sheath, in this work it is called the Chodura layer, where the ions enter with a velocity V_{par} parallel to the magnetic field equal to the sound speed V_S. After this layer, there is the sheath, a collision-less charged region that begins when the component of the positive ion velocity perpendicular to the wall V_x reaches the sound speed V_S. In the Chodura layer, the electric field grows causing the velocity of the positive ions to deviate from the magnetic-field lines.

In this work the magnetized plasma-wall transition is studied by a one-dimensional fluid model, where the continuity and momentum exchange equation are valid for the ions, while the Boltzmann relation is assumed for the electrons. It is shown that the solutions of the model are very sensitive to boundary conditions and selected parameters.

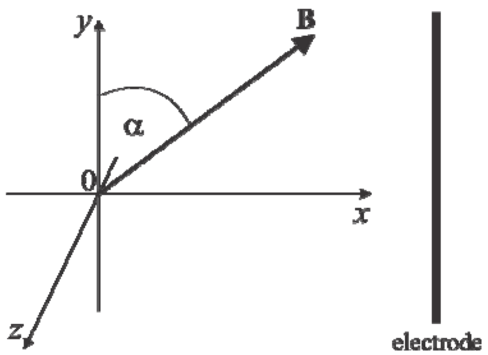

Figure 1: Schematic of the coordinate system.

2. Model

The model is based on continuity (1) and momentum exchange equation (2) for ions, while the Boltzmann relation (3) is assumed for the electrons. In this work only the region up to the sheath edge is considered, so plasma neutrality (4) is assumed.

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = S_i, \tag{1}$$

$$m_i n_i \left(\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i \cdot \nabla)\mathbf{u}_i \right) = n_i e_0 (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla p_i + \mathbf{A}_i - m_i \mathbf{u}_i S_i, \tag{2}$$

$$n_e(\mathbf{r}) = n_0 \exp\left(\frac{e_0 \Phi(\mathbf{r})}{kT_e} \right), \tag{3}$$

$$n_i = n_e. \tag{4}$$

Here m_i is the ion mass, k is the Boltzmann constant, T_e is the electron temperature, Φ is the potential, t is time, $\mathbf{u}_i$ is the ion fluid velocity ($\mathbf{u}_i = (u_x, u_y, u_z)$), n_i is the ion density, n_e is the electron density, n_0 is the plasma density in the unperturbed region far away from the electrode, e_0 is the elementary charge, $\mathbf{E}$ is electric field, $\mathbf{B}$ is magnetic field, p_i is the ion pressure, S_i is the source term and $\mathbf{A}_i$ is the collision term. The ions can exchange momentum in elastic collisions with other particle species, like electrons or neutrals. It is beyond the scope of this paper to analyse the ion collision processes in detail. So the rate of change of momentum of the ions because of such collisions is simply written as:

$$\mathbf{A}_i = -m_i n_i \nu \mathbf{u}_i. \tag{5}$$

Here ν is the ion collision frequency. The source term S_i must be assumed to be a known function. It is reasonable to assume the simplest possible form of such function. One possibility is the constant source term:

$$S_i = \frac{n_0}{\tau}. \tag{6}$$

Here τ is the ionization time, which gives the rate of creation of ion-electron pairs by some ionization mechanism. But in this work the zero source term

$$S_i = 0, \quad (7)$$

is used. The ions are assumed to be isothermal and using the ideal gas law the gradient pressure term is written as $\nabla p_i = kT_i \nabla n_i$. An infinitely large planar electrode is considered. The electrode is perpendicular to the x axis. The magnetic field lies in the xy plane, as shown in Figure 1. The components of the magnetic field are therefore given by $\mathbf{B} = B(\sin\alpha, \cos\alpha, 0)$. Our model is one dimensional. This means that in our model the electric field only has one component: $\mathbf{E} = (E_x, 0, 0)$, related to the potential Φ by:

$$E_x = -\frac{\mathrm{d}\Phi}{\mathrm{d}x}. \quad (8)$$

In addition gradient and Laplace operators are replaced by derivatives over x:

$$\nabla \to \mathbf{e}_x \frac{d}{dx}, \ \nabla^2 \to \frac{d^2}{dx^2}. \quad (9)$$

The steady state is considered, so time derivatives in (1) and (2) are omitted. It is assumed that the zero source term (7) is valid. Using the coordinate system described in Fig. 1 and above assumptions, the system (1) - (4), (7) is written in the following form:

$$\frac{d\Psi}{dX} = -\frac{1}{V_x}\frac{dV_x}{dX}, \quad (10)$$

$$V_x \frac{\mathrm{d}V_x}{\mathrm{d}X} + \frac{\mathrm{d}\Psi}{\mathrm{d}X}(1+\Theta) + A_2 \cos\alpha V_z + A_1 V_x Z = 0, \quad (11)$$

$$V_x \frac{\mathrm{d}V_y}{\mathrm{d}X} - A_2 \sin\alpha V_z + Z A_1 V_y = 0, \quad (12)$$

$$V_x \frac{\mathrm{d}V_z}{\mathrm{d}X} - A_2 \cos\alpha V_x + A_2 \sin\alpha V_y + Z A_1 V_z = 0. \quad (13)$$

The following variables have been introduced:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{n_0 e_0^2}}, \ c_0 = \sqrt{\frac{kT_e}{m_i}}, \ \omega_c = \frac{e_0 B}{m_i} = \frac{2\pi}{T_{cyc}},$$

$$K = \omega_c \tau, \ L = c_0 \tau, \ \mu = \frac{m_e}{m_i}, \ L_c = \frac{c_0}{\nu} = c_0 T_{coll},$$

$$r_{Li} = \frac{c_0}{\omega_c}, \ n = \frac{n_i}{n_0}, \ \Psi = \frac{e_0 \Phi}{kT_e}, \ \Theta = \frac{T_i}{T_e}, \quad (14)$$

$$\varepsilon = \frac{\lambda_D}{L}, \ Z = \nu\tau = \frac{\tau}{T_{coll}} = \frac{L}{L_c},$$

$$V_x = \frac{u_x}{c_0}, \ V_y = \frac{u_y}{c_0}, \ V_z = \frac{u_z}{c_0}, \ X = \frac{x}{\lambda_D} \ \text{or}$$

$$X = \frac{x}{L} \ \text{or} \ X = \frac{x}{r_{Li}} \ \text{or} \ X = \frac{x}{L_c}.$$

From (14) it can be seen easily that:

$$K = \frac{L}{r_{Li}} = \frac{2\pi\tau}{T_{cyc}}, \ \frac{K}{Z} = \frac{L_c}{r_{Li}} = \frac{2\pi T_{coll}}{T_{cyc}},$$
$$K\varepsilon = \frac{\lambda_D}{r_{Li}}, \ Z\varepsilon = \frac{\lambda_D}{L_c}. \quad (15)$$

Here ε_0 is the permittivity of the free space, ω_c is the angular cyclotron frequency of the ions and T_{cyc} is the cyclotron period. The potential Φ is normalized to the electron temperature divided by elementary charge kT_e/e_0. The components of the ion velocity are normalized to the so called normalizing velocity c_0, defined in (14). The problem is characterized by 4 characteristic length scales: the ionization length L, the Debye length λ_D, the mean free path or collision length L_c and the ion Larmor radius r_{Li}. The ionization length L is the distance that an ion moving with the normalizing velocity c_0 passes in one ionization time τ. The mean free path L_c is the distance that an ion moving with the normalizing velocity c_0 passes in the so called collision time $T_{coll} = 1/\nu$. An ion that enters the magnetic field B with normalizing velocity c_0 perpendicularly to the magnetic field line gyrates with the Larmor radius r_{Li}. Obviously the space coordinate x can be normalized to any of these lengths.

A remark should be given about the ion sound velocity c_S. The ion sound velocity c_S is defined as:

$$c_S = \sqrt{\frac{kT_e^* + \kappa k T_i}{m_i}}. \quad (16)$$

Here T_e^* is the so called screening temperature [5]. In our case it is equal to T_e because the electrons are the only negatively biased particle species. Using (14) and taking into account that $\kappa = 1$, because the ions are isothermal, the ion sound velocity (16) is written as:

$$V_S = \frac{c_S}{c_0} = \sqrt{1+\Theta}. \quad (17)$$

The coefficients A_1 and A_2 in (10) - (13) depend on the normalization of the space coordinate x and they are given in Table 1.

Table 1. The coefficients A_1 and A_2 for various normalizations of the space coordinate x.

Symbol	$X = x/L$	$X = x/\lambda_D$	$X = x/L_c$	$X = x/r_{Li}$
A_1	1	ε	$1/Z$	$1/K$
A_2	K	$K\varepsilon$	K/Z	1

Once the system (10) - (13) is solved and the velocity $\mathbf{V} = (V_x, V_y, V_z)$ is found, it is convenient to define also the parallel velocity V_{par} and the angle of incidence β in the following way:

$$V_{par} = \frac{\mathbf{V}\cdot\mathbf{B}}{B} = V_x \sin\alpha + V_y \cos\alpha, \qquad (18)$$

$$\beta = \arctan\left(\frac{V_x}{V_y}\right). \qquad (19)$$

3. Results

We now present some results of the model presented in the previous section. In order to find the unknown functions $\Psi(X)$, $V_x(X)$, $V_y(X)$ and $V_z(X)$ from the system (10) - (13) first the normalization of x must be selected and then the parameters K, Z, α, Θ and ε must also be selected. In the next step boundary conditions must be selected. From Figure 1 it can be seen that the numerical integration of the equations starts at $X = 0$ and proceeds in the positive direction of X towards the electrode, so strictly mathematically speaking, one is dealing with initial value problem. The "initial" conditions (since X is the space coordinate these are in fact *boundary conditions*) that are usually selected are the following:

$$\Psi(0) = 0,\ \ V_x(0) = V_0,\ V_y(0) = 0,\ V_z(0) = 0. \ (20)$$

The first boundary condition simply tells that the plasma potential at $X = 0$ is selected as the zero of the potential. In our model the ions are born at rest so all three velocity components should be zero. But in this case only the zero solution of the system can be found. So a small starting velocity V_0 in positive X direction must be selected. In Figure 2 it is illustrated how crucial is the selection of V_0 for the solutions of the system (10) - (13). The following parameters are selected: $K = 100$, $\alpha = 20^o$, $\Theta = 0$ and $Z = 0.05$. The space coordinate is normalized to L, so $A_1 = 1$ and $A_2 = K$ are taken from Table 1. Only the ion velocity V_x is shown versus X. In addition also the velocity V_{par} (formula (18)) is shown. A thin horizontal line marks the ion sound velocity V_S. Four different values of V_0 are selected. They are written in the plots. A relatively small change in V_0 causes a big change in the profiles $V_x(X)$ and $V_{par}(X)$. Except for the case with $V_0 = 0.159$ the profiles $V_x(X)$ and $V_{par}(X)$ exhibit regular oscillations with slightly increasing trend. When V_x reaches the ion sound velocity V_S, the sheath edge is reached. The system (10) - (13) becomes singular and the computation breaks down. As V_0 is increased the number of periods of the oscillation decreases and the period becomes slightly longer.

In Figure 3 similar results are shown. The following parameters are selected: $K = 100$, $\alpha = 20^o$,

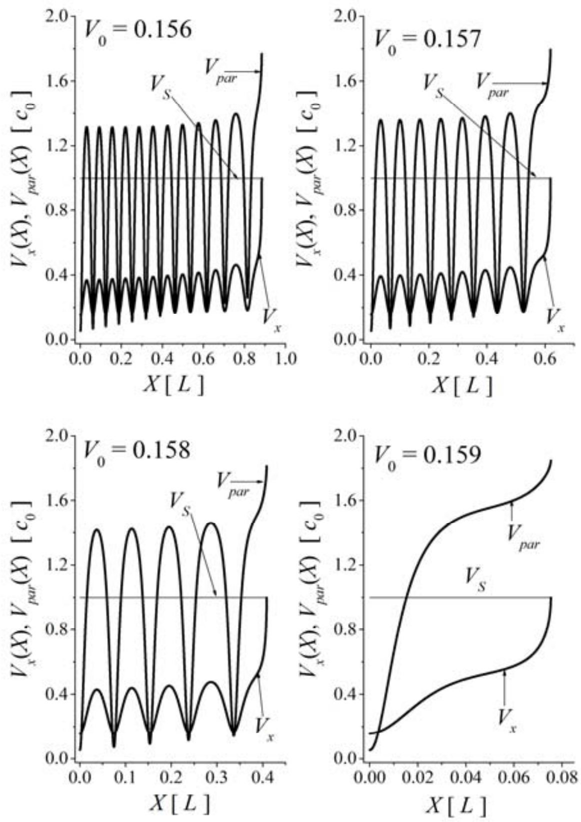

Figure 2: Solutions of the system (10) - (13) for the parameters $K = 100$, $\alpha = 20^o$, $\Theta = 0$ and $Z = 0.05$. The space coordinate is normalized to L. Four different values of V_0 are selected. The values are marked in the plots.

$\Theta = 0$ and $V_0 = 0.159$. The space coordinate is normalized to L. Four different values of Z are selected and they are given in the plots. Very similar oscillating solutions are observed. As Z is increased the number of oscillations decreases and the period slightly increases. At some point V_x reaches the ion sound velocity V_S, and the integration of the system (10) - (13) breaks down. The rate of elastic collisions Z and boundary velocity V_0 have a very similar role.

In the last figure (Fig. 4) an example of the nonoscillatory solution is shown. The following parameters are selected: $K = 100$, $\alpha = 20^o$, $\Theta = 0$, $V_0 = 0.159$ and $Z = 0.02$ are selected. In plot (*a*) the electric field profile $\Psi(X)$ is shown, in figure (*b*) the corresponding electric field profile $-d\Psi/dX$ is presented, in graph (*c*) the velocity profiles $V_x(X)$ and $V_{par}(X)$ are displayed and in figure (*d*) the profile of the angle of incidence $\beta(X)$ (formula (19)) is presented. Since $V_x(0) = 0.159$ and $V_y(0) = 0$ is selected, the ions start to flow in the direction

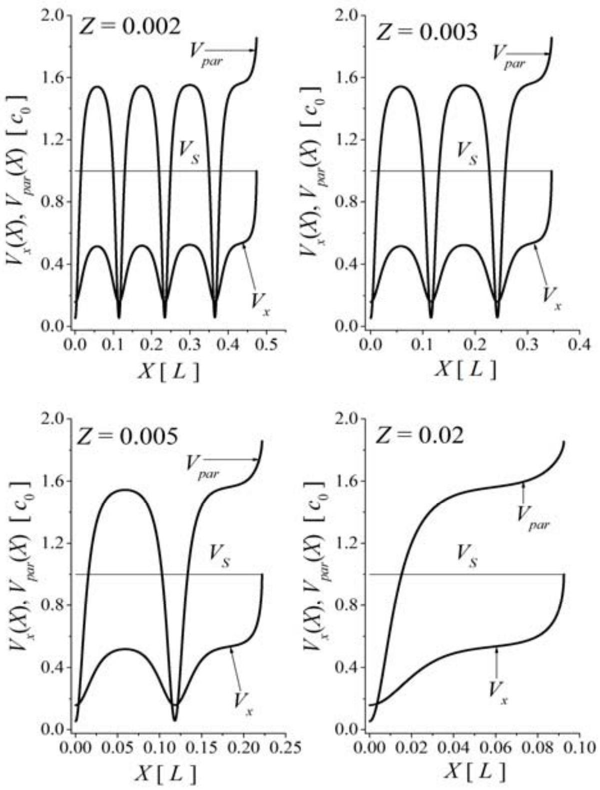

Figure 3: Solutions of the system (10) - (13) for the parameters K = 100, α = 20°, Θ = 0 and V_0 = 0.159. The space coordinate is normalized to L. Four different values of Z are selected. The values are marked in the plots.

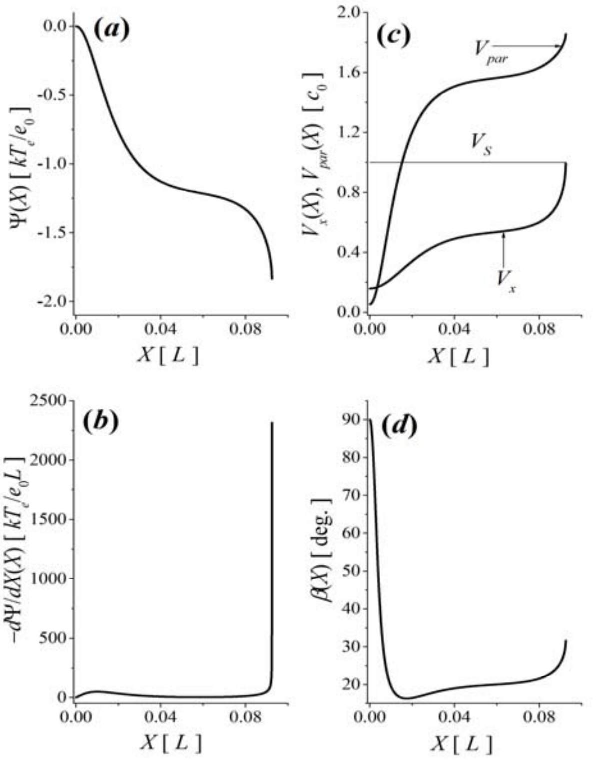

Figure 4: Solutions of the system (10) - (13) for the parameters K = 100, α = 20°, Θ = 0, V_0 = 0.159 and Z = 0.02.

perpendicular to the electrode, but then they are quickly deviated in the direction of the magnetic field lines. Only very close to the sheath edge they start to turn in the direction of the electric field.

4. Conclusions

A one-dimensional fluid model of the plasma-wall transition has been presented. The problem has been analysed in the pre-sheath scale only [2,3], since quasi-neutrality has been assumed a-priori. It has been shown that for certain parameters the solutions of the model exhibit regular and rather high amplitude oscillations. The solutions are very sensitive to the boundary ion velocity V_0. The oscillations are strongly damped by elastic ion collisions. At present it is not yet clear, whether the observed oscillations are a consequence of some plasma instability or are they a numerical effect. Such behaviour of the solutions is a consequence of absence of any ionization – formula (7). Oscillatory solutions magnetized plasma-wall transition problem have been reported in the literature [4,5], but never pointed out as a problem that needs additional research and clarification. In this work the oscillating solutions of the model are addressed as an open question but additional work will be necessary to get some answers.

Acknowledgements

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