Modeling of laser produced plasma expansion into vacuum with kappa distributed electrons

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The expansion of semi-infinite laser plasma into vacuum is analyzed with a hydrodynamic model for cold ions assuming kappa distributed electrons. Similarly to Mora study of a plasma expansion into vacuum [Mora P., Phys. Rev. Lett. **90**, 185002 (2003)], we formulated empirical expressions for the electric field strength, velocity and position of the ion front in isothermally expanding plasma. Analytic expressions for the maximum ion energy and the spectrum of the accelerated ions in the plasma were derived and discussed.

1. Introduction

The dynamics of the ion front in laser plasma expansion is still not completely resolved. The most famous work on this subject is the model proposed in 2003 by Mora [1], who presented a detailed theory of the self-similar, quasineutral expansion driven by Maxwellian electrons, including the charge separation effects on the scale length of the electron Debye length at the ion front. By analogy with this study, we calculated self-similar solution for one-dimensional nonrelativistic, collisionless isothermally expanding plasma with a population of kappa distributed electrons, taking into account charge separation effects.

2. Governing model equations

Electron kappa distribution implies the spatial electron density to have the form [2]:

$$n_e = n_{e0}\left(1-\Phi/\left(\kappa-\tfrac{3}{2}\right)\right)^{-\kappa+1/2} \qquad (1)$$

$\Phi = e\varphi/T_e$ is the normalized electric potential.

The cold ion motion is modeled by fluid equations:

$$\frac{\partial n_i}{\partial t} + v_i\frac{\partial(n_i v_i)}{\partial x} = 0 \qquad (2)$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} + \frac{e}{m_i}\frac{\partial \varphi}{\partial x} = 0 \qquad (3)$$

$$\varepsilon_0\frac{\partial^2\varphi}{\partial x^2} = e\left(n_e - n_i\right) \qquad (4)$$

3. Self-similar solution

When charge quasineutrality is assumed, a self-similar solution of Eqs. (2) and (3) can be constructed by using $\tilde{n}_i = n_i/n_{io}$, $\tilde{v}_i = v_i/c_s$, c_s is the ion sound velocity and n_{io} is the initial density of the plasma [3]. By using the dimensionless self-similar variable $\xi = x/c_s t$, we obtain for the ions:

$$\Phi_{ss} = -\frac{1}{(\kappa-1)^2}\left(\begin{array}{l}\frac{1}{4}\left(\kappa-\frac{1}{2}\right)\xi^2 + \\ \left(\kappa-\frac{1}{2}\right)^{3/2}\left(\kappa-\frac{3}{2}\right)^{1/2}\xi + \left(\kappa-\frac{3}{4}\right)\left(\kappa-\frac{3}{2}\right)\end{array}\right) \qquad (5)$$

$$\tilde{v}_{i,ss} = \left(\frac{1}{\kappa-1}\right)\left(\left(\kappa-\tfrac{1}{2}\right)\xi + \sqrt{\left(\kappa-\tfrac{1}{2}\right)\left(\kappa-\tfrac{3}{2}\right)}\right) \qquad (6)$$

$$\tilde{n}_{i,ss} = \left(\frac{1}{2}\frac{\sqrt{\left(\kappa-\frac{1}{2}\right)}}{(\kappa-1)\sqrt{\left(\kappa-\frac{3}{2}\right)}}\xi + \frac{\left(\kappa-\frac{1}{2}\right)}{(\kappa-1)}\right)^{-2\kappa+1} \qquad (7)$$

The self-similar solution has no meaning as long as the initial local Debye length, λ_D is larger than the ion density scale length l_{ss}. By evaluating l_{ss} from (7) at x=0, one finds $l_{ss} = c_s t\sqrt{\kappa-3/2}/\sqrt{\kappa-1/2}$. Consequently, λ_D exceeds l_{ss} at the front position:

$$x_f/c_s t = 2\sqrt{\left(\kappa-\tfrac{3}{2}\right)}\left(\frac{(\kappa-1)}{\sqrt{\left(\kappa-\frac{1}{2}\right)}}\left(\frac{\sqrt{\left(\kappa-\frac{1}{2}\right)}}{\sqrt{\left(\kappa-\frac{3}{2}\right)}}\frac{1}{\omega_{pi}t}\right)^{\frac{1}{-\kappa+1/2}} - \sqrt{\left(\kappa-\tfrac{1}{2}\right)}\right) \qquad (8)$$

ω_{pi} is the ion plasma frequency.

Taking the time derivative of (6) and using (8), the electric field at the ion front for $\omega_{pi}t >> 1$ is:

$$\frac{E_f(t)}{e/m_i} = 2\frac{\sqrt{\left(\kappa-\frac{3}{2}\right)}}{\sqrt{\left(\kappa-\frac{1}{2}\right)}}\left(\frac{\sqrt{\left(\kappa-\frac{1}{2}\right)}}{\sqrt{\left(\kappa-\frac{3}{2}\right)}}\right)^{\frac{1}{-\kappa+1/2}}\omega_{pi}\left(\omega_{pi}t\right)^{\frac{-\kappa+3/2}{\kappa-1/2}} \qquad (9)$$

When $\kappa \longrightarrow \infty$, we retrieve the maxwellian case [1].

3.1 The Electric field at the ion front

In order to express the electric field at all instants of time, we propose the generalized expression:

$$E(t) = \frac{E(t=0)}{\left[1+F(\kappa)\left(\omega_{pi}t\right)^2\right]^{\frac{\kappa-3/2}{2\kappa-1}}} \qquad (10)$$

with $E(t=0) = \sqrt{2}E_0\left(\frac{\kappa-3/2}{\kappa-1/2}\right)^{\kappa/2-3/4}$

Taking account of the two limits, we propose a relation for $F(\kappa)$ which is suitable for numerical values of the electrical field in the following form:

$$F(\kappa)=\left(\frac{1}{2}\right)^{\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}}\left(\frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{\kappa-3/2-\frac{2}{\kappa-3/2}-\frac{2}{k}} \qquad (11)$$

In Fig. 1, it was shown that by increasing the population of superthermal electrons (κ decreasing), the resulting electric field would be stronger. As the population of energetic electrons increases, the charge separation process takes place more efficiently and larger electric field is set up.

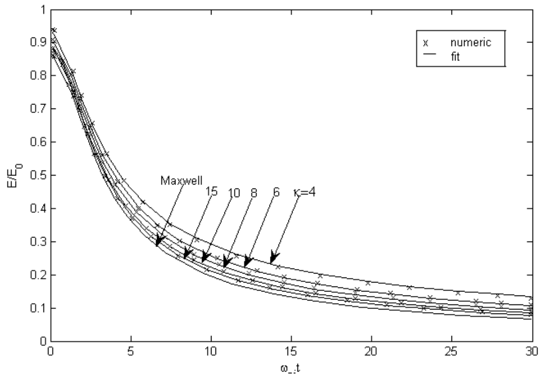

FIG. 1. Electric field as a function of time for different κ .

3.2 Velocity and position of the ion front

By integrating Eq. (10), we obtain, the normalized ion front velocity as function of time,

$$v_i(t)/c_s = \sqrt{2}\left(\frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{(\kappa/2-3/4)}\omega_{pi}t\ {}_1^2F\left(\left[1/2,(\kappa-3/2)/(2\kappa-1)\right],[3/2],-F(\kappa)\left(\omega_{pi}t\right)^2\right) \qquad (12)$$

and the normalized ion position by integrating (12),

$$x_i(t)/\lambda_D = \frac{\left(\kappa-\frac{1}{2}\right)}{F(\kappa)\left(\kappa+\frac{1}{2}\right)}\sqrt{2}\left(\frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{(\kappa/2-3/4)}{}_1^2F\left(\left[-1/2,-(\kappa+1/2)/(2\kappa-1)\right],[1/2],-F(\kappa)\left(\omega_{pi}t\right)^2\right) \qquad (13)$$

where ${}_1^2F$ is the Gaussian hypergeometric function.

As a result, when the population of nonthermal electrons is increasing (κ decreasing), the velocities (Fig. 2) and the positions of the ion front (Fig. 3) are increasing which means that the expansion took place faster and the ions were accelerated to higher energies comparatively to the maxwellian case.

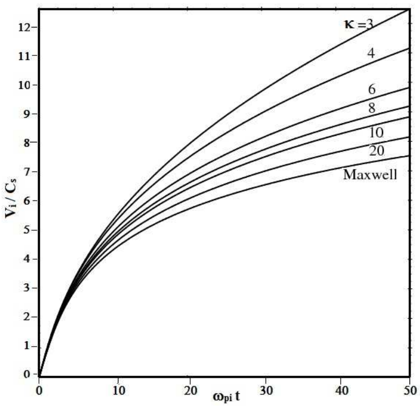

FIG. 2 Time dependence ion velocity for different κ

3.3 Energy spectrum and maximum ion energy

Ion energy spectrum in the self-similar model is:

$$\frac{dN}{d\varepsilon}/\left(n_{i0}\lambda_{D0}/T_e\right)=\frac{\omega_{pi}t}{\sqrt{2(\varepsilon/\varepsilon_0)}}\left[1+\frac{0.5}{\sqrt{(\kappa-1.5)(\kappa-0.5)}}\sqrt{2(\varepsilon/\varepsilon_0)}\right]^{-2\kappa+1} \qquad (14)$$

N is the ion number per unit energy and per unit surface. The energy $\varepsilon = mv_i^2/2$ and $\varepsilon_0 = T_e$.

$\varepsilon_{max} = \frac{1}{2} m_e v_{max}^2$ is the cutoff ion energy where v_{max} is deduced from Eq. 12 and Fig. 2.

In Fig. 4, it is shown that the number of ions per energy unit and the maximum ion energy are increasing, with increasing of the population of nonthermal electrons.

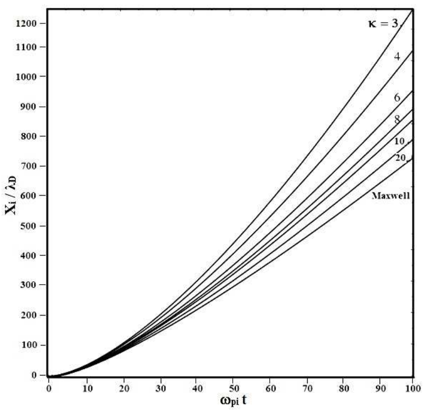

FIG. 3 Time dependence of ion position for different κ

4. Conclusion

In this work, it was shown that the plasma expansion took place faster and farther from the interface plasma-vacuum, when the population of energetic electrons is increasing. The maximum ion energy is also increasing and the energetic ions will be more populated due to stronger electric field.

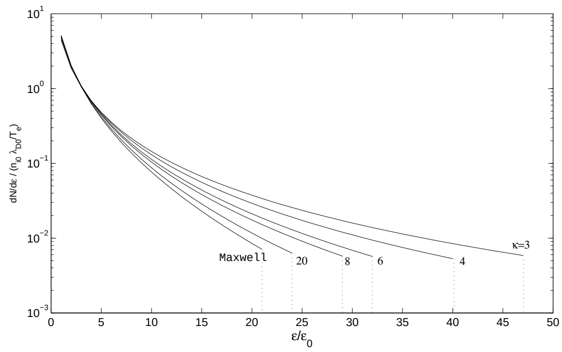

FIG. 4: Energy spectrum at $\omega_{pi}t = 30$ for different κ

5. References

[1] P. Mora, Phys. Rev. Lett. **90**, 185002 (2003)

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[3] H. Schamel, Phys. Scr**.** **20**, 306 (1979)