Theoretical study of plasma sustained around dielectric cylinder by travelling electromagnetic wave

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Gas discharge can be produced and sustained by travelling electromagnetic waves in various geometries: planar, spherical, cylindrical and coaxial. There are both experimental and theoretical indications, that plasma can be produced around dielectric cylinder. In order to find out these conditions theoretically we have built one-dimensional fluid model for vacuum–plasma and dielectric–plasma configurations. The basic relation in our model for describing wave propagation characteristic is the local dispersion relation obtained from Maxwell's equations. The surface-wave-discharge is axially and radially inhomogeneous, so the local dispersion relation gives the so called phase diagrams – the dependence between the normalized plasma density and the dimensionless wave number at different wave modes. Analyzing the phase diagrams, the axial profile of dimensionless plasma density and the 3D distribution of electric and magnetic field components, one can obtain information about the ability of the electromagnetic wave to sustain plasma at given discharge conditions.
The purpose of this work is to investigate theoretically the behaviour of wave phase diagrams and axial profiles at various discharge conditions and to find out the values of plasma parameters at which plasma can be sustained.

1. Introduction

Electromagnetic wave travelling along the dielectric-gas interface can produce plasma. These gas discharges called surface-wave-discharges (SWDs) exist in various geometries: planar, spherical, cylindrical, coaxial, and around dielectric cylinder. When at the dielectric tube axis is arranged a metal rod, the plasma is produced outside the tube which is the typical coaxial surface-wave-sustained discharge (CSWD). The experimental investigations show that in some conditions the plasma can be produced outside the dielectric cylinder even when there is not any metal antenna at the dielectric axis [1].

In order to find out these conditions theoretically we have built one-dimensional fluid model for vacuum–plasma and dielectric–plasma configurations. The basic relations in our model are the local dispersion relation describing the wave propagation and the wave energy balance equation, both obtained from Maxwell's equations. Analyzing the phase diagrams, the axial profile of dimensionless plasma density and the 3D distribution of electric and magnetic field components, one can obtain information about the ability of the electromagnetic wave to sustain plasma at given discharge conditions.

The purpose of this work is to investigate theoretically the behavior of wave phase diagrams and axial profiles at various discharge conditions and to find out the values of plasma parameters at which plasma can be sustained.

2. Modelling

We investigate the possibility for electromagnetic surface-waves to create and sustain plasma using one-dimensional fluid model. The propagating electromagnetic surface wave creates the discharge and also the discharge is propagation medium for the wave at the same time.

The propagation of the electromagnetic wave is described by the Maxwell's equations. After solving the Maxwell's equations we obtain the wave equation and we apply boundary conditions for continuity of the wave field tangential components to find the local dispersion equation and by solving it we obtain phase diagrams - dependence between the normalized plasma density N (ω / ω_p respectively) and dimensionless wave number k_zR.

We assume that the electromagnetic wave field components have the form:

$$E^{p,v}_{r,\phi,z}(r,\phi,z,t) = \mathrm{Re}\left\{F^{p,v}_{r,\phi,z}(r,z)E(z)\exp\left[-i\omega t + im\phi + i\int_{z_0}^{z} dz'k(z')\right]\right\}$$

$$B^{p,v}_{r,\phi,z}(r,\phi,z,t) = \mathrm{Re}\left\{G^{p,v}_{r,\phi,z}(r,z)E(z)\exp\left[-i\omega t + im\phi + i\int_{z_0}^{z} dz'k(z')\right]\right\}$$

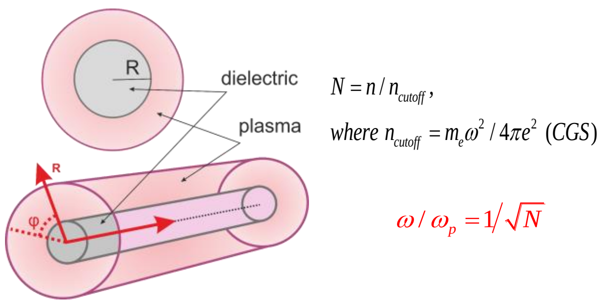
The solutions of the wave equation give the wave field components amplitude as combinations of Bessel or modified Bessel functions:

$$F_z^{\mathrm{v}}(a_{\mathrm{v}}\rho) = C_1 I_0(a_{\mathrm{v}}\rho) + C_2 K_0(a_{\mathrm{v}}\rho)$$
$$F_z^{\mathrm{d}}(a_{\mathrm{d}}\rho) = \begin{cases} C_3 I_0(a_{\mathrm{d}}\rho) + C_4 K_0(a_{\mathrm{d}}\rho) & x > \sigma\sqrt{\varepsilon_{\mathrm{d}}} \\ C_3' J_0(a_{\mathrm{d}}\rho) + C_4' N_0(a_{\mathrm{d}}\rho) & x < \sigma\sqrt{\varepsilon_{\mathrm{d}}} \end{cases} \quad (1)$$
$$F_z^{\mathrm{p}}(a_{\mathrm{p}}\rho) = C_5 I_0(a_{\mathrm{p}}\rho)$$

where

$$a_{\mathrm{p}} = (x^2 - \sigma^2\varepsilon_{\mathrm{p}})^{1/2}, \quad a_{\mathrm{d}} = (x^2 - \sigma^2\varepsilon_{\mathrm{d}})^{1/2}, \quad a_{\mathrm{v}} = (x^2 - \sigma^2)^{1/2}$$
$$\rho = \frac{r}{R}, \quad x = kR, \quad \sigma = \frac{\omega R}{c}$$

At low pressures plasma can be considered as a weakly dissipative medium. At this assumption the ratio of the electron-neutral collisions frequency ν and the wave angular frequency ω is smaller than unity and can be neglected in the plasma permittivity expression, i.e. we can use the simple form $\varepsilon_{\mathrm{p}} = 1 - \omega_{\mathrm{p}}^2/\omega^2$ ($\omega_{\mathrm{p}} = (4\pi e^2 n/m_e)^{1/2}$ (CGS) being the plasma frequency). The coaxial structure is investigated on the base of one-dimensional fluid



ω – external wave frequency;
k_z – wave number;
m – azimuthal wave number;
R – radius of plasma (outer diameter of dielectric cylinder);
ε_{d} – dielectric permittivity (for vacuum - plasma configuration $\varepsilon_{\mathrm{d}} = 1.0$);
N – normalized plasma density;

$\sigma = \dfrac{\omega R}{c}$ – plasma parameter;

Figure 1. Investigated coaxial geometry and modeling parameters

model. The plasma is produced and sustained by a surface electromagnetic wave, which propagates along the interface between the plasma and the dielectric tube or vacuum.

It is difficult to present the dispersion equation in a compact form so we are presenting it now in more general form:

$\mathrm{D}(m, \sigma, k, \omega, \omega_{\mathrm{p}}) = 0.$ (2)

3. Results

Investigated SW propagation is for discharges around dielectric cylinder varying the geometrical (dielectric radius R) and dielectric (permittivity ε_{d}) for different wave modes (m = 0 – azimuthally symmetric wave, m = 1 – dipolar wave mode, m = 2 – quadrupolar wave mode).

For $f = const$ = 2.45 GHz plasma parameter σ can be regarded as plasma radius R hence $\sigma = 0.1 \rightarrow R \approx 2$ mm; $\sigma = 0.5 \rightarrow R \approx 10$ mm; $\sigma = 1.5 \rightarrow R \approx 29$ mm; $\sigma = 2.0 \rightarrow R \approx 40$ mm.

The values ε_{d} of dielectric permittivity are selected in such a manner as to cover the range of the most - commonly used dielectric materials (ε_{d} = 2.1 – PTFE; ε_{d} = 2.25 - Polyethylene; ε_{d} = 3.9 – quartz, ε_{d} = 4.7 – glass).

Example of one the phase diagrams for configuration vacuum-plasma is presented on the next figure (figure 2). Dotted lines represent the

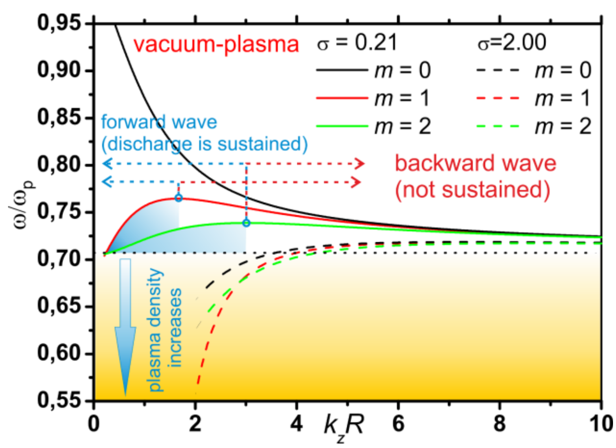


Figure 2. Consideration of phase diagrams for sustainability of discharge

trend of phase diagrams and that value corresponds to resonant plasma density. That value is respectively $N_{\mathrm{res}} = 1 + \varepsilon_{\mathrm{v}} = 2$ (resonance frequency $(\omega/\omega_{\mathrm{p}})_{\mathrm{res}} = 1/\sqrt{2}$) for vacuum and in the general case $(\omega/\omega_{\mathrm{p}})_{\mathrm{res}} = \dfrac{1}{\sqrt{1+\varepsilon_{\mathrm{d}}}}$ for dielectric with permittivity ε_{d}.

As shown each phase curve reaches a maximum value, passing or not (depending on specific conditions) over the resonant plasma frequency. On this basis, we can consider each phase curve to be composed of two areas:

- area of forward wave propagation of direct wave: when the phase and group velocity of propagating wave have the same sign. This corresponds to the range of wave numbers, for which the plasma density decreases and reaches a minimum value;

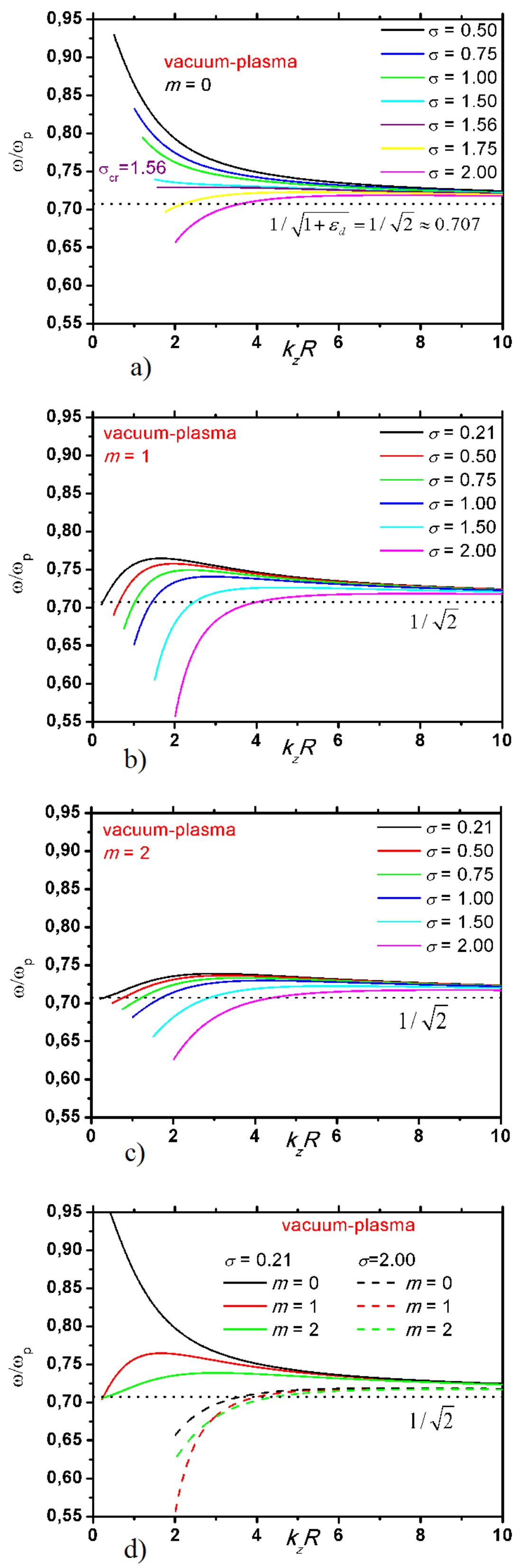


Figure 3. Phase diagrams for the vacuum-plasma configuration in various wave modes: **a)** m = 0 – azimuthally symmetric wave mode, **b)** m = 1 – dipolar WM, **c)** m = 2 – quadrupolar WM and **d)** comparison for different plasma parameter values

- area of backward wave propagation of direct wave: when the phase and group velocity of propagating wave have opposite signs. This corresponds to a range of wave numbers for which the plasma density increases (in cases tends to resonant plasma density).

We assume that the backward wave cannot sustain plasma and therefore the maximum in the phase curve corresponds to the end of the plasma column in the axial direction.

Results of modelling for vacuum-plasma configuration are presented on figure 3. Calculation are performed for m = 0 – figure 3a), m = 1 – figure 3b), m = 2 – figure 3c) and different plasma parameter values σ ($\sigma = 0.5 \rightarrow R \approx 10$ mm to $\sigma = 2.0 \rightarrow R \approx 40$ mm).

Plasma density is strongly influenced by value of σ (radius R). Increase of plasma parameter results in shifting of phase diagrams towards higher plasma density and forward wave propagation, which can sustain the discharge. For m = 0 diagrams are divided into two areas (FW and BW propagation) by critical value σ = 1.56. For higher wave modes m = 1, 2 such trend is not observed (no critical value). At bigger radiuses (σ = 2.00), higher wave modes m = 1, 2 are better at sustaining plasma than azimuthally symmetric mode m = 0.

To investigate the influence of propagation medium simulations were performed for common values of dielectric permittivity (ε_d = 2.1 – PTFE; ε_d = 2.25 - Polyethylene; ε_d = 3.9 – quartz, ε_d = 4.7 – glass). For different dielectric materials (higher dielectric permittivity ε_d) trend is similar to that observed for vacuum. Higher dielectric permittivity results into higher plasma density. Varying σ for different ε_d values, a relation between σ and ε_d can be found (figure 5).

For smaller values of σ quadrupolar wave produces plasma with higher density than dipolar one. For m = 1 and 2 one can see forward and backward wave propagation regions while for m = 0 only backward wave is observed. Increasing σ affect to phase diagrams. Azimuthally symmetric wave is able to sustain discharge only at bigger radiuses (σ = 2.00 → $R \approx 40$ mm).). Dipolar wave may produce plasma with the highest density, while wave with m = 2 is able to sustain and produce plasma with the lowest density.

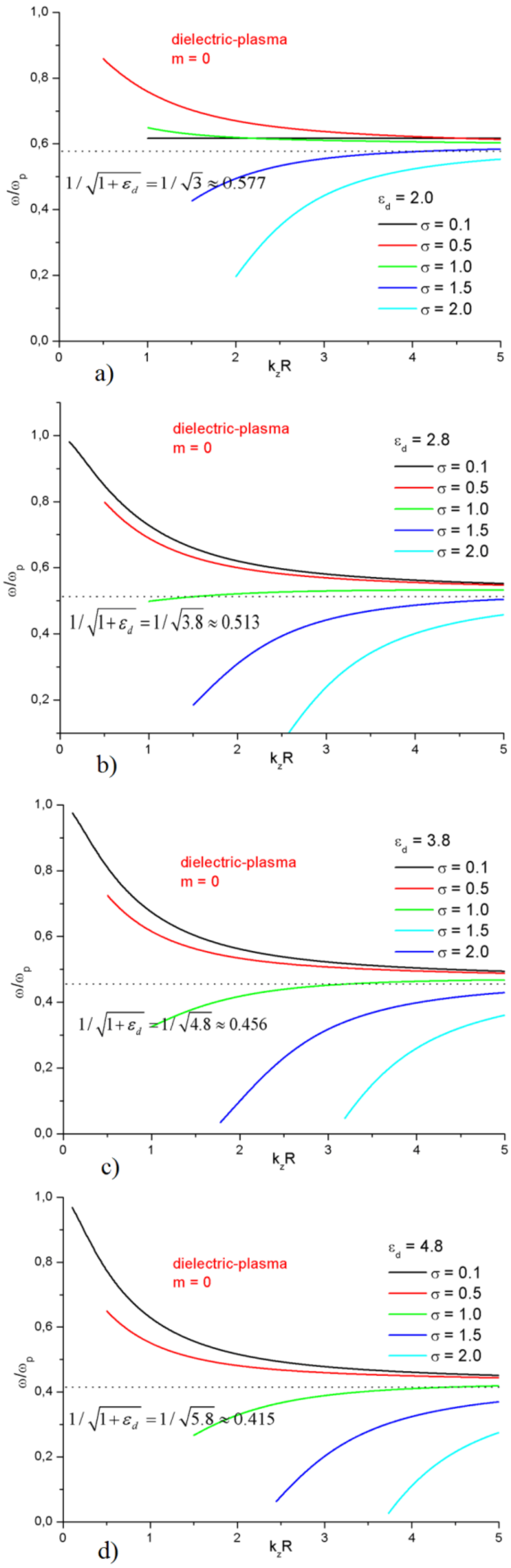


Figure 4. Phase diagrams for the dielectric–plasma configuration for various values of dielectric permittivity **a)** ε_d = 2.0, **b)** ε_d = 2.8, **c)** ε_d = 3.8 **d)** ε_d = 4.8

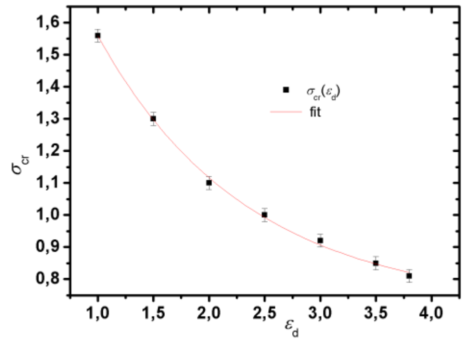


Figure 5. Dependence of the critical plasma parameter of the dielectric permittivity

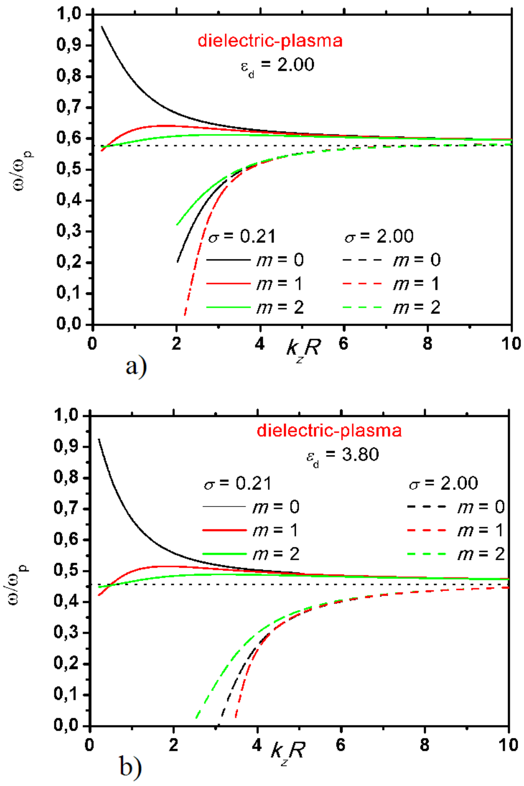


Figure 6. Comparison of phase diagram trends for two values of permittivity **a)** ε_d = 2.0, **b)** ε_d = 3.8

4. References

[1] Liang R., Nie Z., Liang B., Liang Y., Chang X., He L., Li Z. MICROWAVE DISCHARGES: Fundamental and Applications, Ed. M. Kando and M. Nagatsu, Japan: 2009, 95.