Drift wave turbulence in laboratory and space plasmas

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Electrostatic drift wave have been observed in laboratory plasmas as well as in nature, the Earth's ionosphere in particular. For weakly unstable plasmas, the waves can appear as coherent while for strongly driven systems a turbulent evolution can be observed. To account for the conditions in the polar ionospheres it will, in general, be necessary to take into account the vertical magnetic field aligned plasma density gradients coupling the drift waves and the gravitational sound waves.

1. Introduction

In nature as well as many laboratory experiments we can have conditions where the equilibrium plasma density has a gradient along as well as perpendicular to an externally imposed stationary magnetic field $\mathbf{B}_0$. In the polar ionospheres, in particular, magnetic flux-tubes with enhanced plasma density are often associated with auroral precipitation and auroral patches [1] where the plasma density has a gradient perpendicular to $\mathbf{B}_0$, as well as a decreasing density along magnetic field lines, i.e. increasing altitudes. The case with plasma density gradients perpendicular to $\mathbf{B}_0$ have previously been studied and the conditions for electrostatic drift wave instabilities been established [2,3]. The basic results are generalized here to allow for a density gradient along the magnetic field as well, using a simple solvable gravitational plasma model [4]. The real as well as the imaginary parts of the dispersion relation of the waves are modified by this vertical gradient. In the standard reference case the instabilities are caused by the plasma resistivity [5] inhibiting the free electron flow that could establish a local isothermal Boltzmann distribution of the electrons. These are the conditions studied mostly in laboratory experiments, such as Q-machines. For ideal collisionless kinetic plasma conditions the most likely source of enhanced growth rates is field aligned electron currents [2,6].

Linearly unstable electrostatic drift waves are seen to develop a strongly turbulent state that can be characterized by power-law spectra. These spectra seem to have universal properties and several similarities between results from laboratory and from space plasmas can be found. In the present summary we show some illustrative results for spectral indexes obtained in laboratory and in space plasmas. In particular, for data obtained by instrumented rockets we find support for arguments favouring universal power laws.

2. Laboratory experiments

The linear complex dispersion relation for electrostatic resistive drift waves can be found as

$$\omega(\omega - \omega_i) + i\sigma_\parallel[\omega - \omega^* + b(\omega - \omega_i)] = 0\,, \quad (1)$$

where we introduced the universal notation $\sigma_\parallel \equiv (k^2_z/k^2_y)(\omega_{ce}\tau_{ei})\Omega_{ci}$, $b \equiv k^2_y a^2_i$ with $a^2_i \equiv T_e/(M\,\Omega^2_{ci})$, the drift frequency $\omega^* = k_y U_{De} = -k_y(n'_0/n_0)T_e/(eB_0)$, while $\omega_i \equiv -\theta\omega^*$ with $\theta \equiv T_i/T_e$ and $n_0 = n_0(x)$ being the unperturbed plasma density with $\mathbf{x} \perp \mathbf{B}_0$. The electron ion collision frequency is $\nu_{ie} \equiv \tau_{ie}^{-1}$, the electron and ion temperatures T_e and T_i, and the electron and ion cyclotron frequencies being ω_{ce} and Ω_{ci}. The ambient homogeneous magnetic field $\mathbf{B}_0$ is in the $\mathbf{z}$-direction. The linear instability of the resistive drift waves is caused by a small phase difference between the potential and the electron density due to the electron-ion collisions that prohibit the free electron flow along magnetic field lines from wave crest to wave trough. The isothermal Boltzmann equilibrium of the electrons is thereby perturbed.

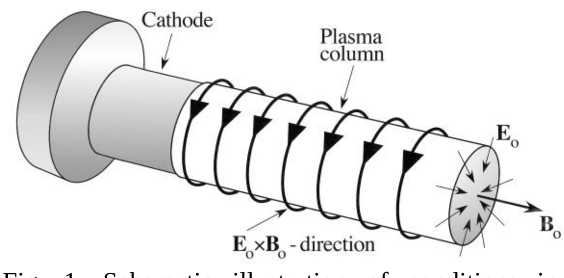

Fig. 1. Schematic illustration of conditions in a strongly magnetized Q-machine plasma, where the $\mathbf{E}_0\times\mathbf{B}_0/B_0^2$ plasma rotation enhances the drift wave fluctuations. The plasma is produced by surface ionization on the hot cathode to the left.

The finite length of many laboratory experiments will not allow very long wavelengths parallel to $\mathbf{B}_0$, so the relevant growth rates will often be small compared to the maximum possible value derived from (1). Plasma rotation (as in Fig. 1) and plasma velocity shear will enhance the growth rates and large amplitude waves can then be observed. In such cases fully developed strongly turbulent conditions can be found with turbulent spectra characterized by well-defined spectral power laws, as shown in Fig. 2. Spectra for potential as well as plasma density fluctuations can be measured [7].

Theoretical studies have suggested that fully developed strong drift wave turbulence can be characterized by power-law spectra. For fluctuations in potential [8], several subranges could be identified, with k^{-3} in a production range and a "coupling" subrange with k^{-5}. The electric field fluctuations in the coupling rage will have a power-law k^{-3}. These results seem in qualitative agreement with a number of laboratory observations [8,9].

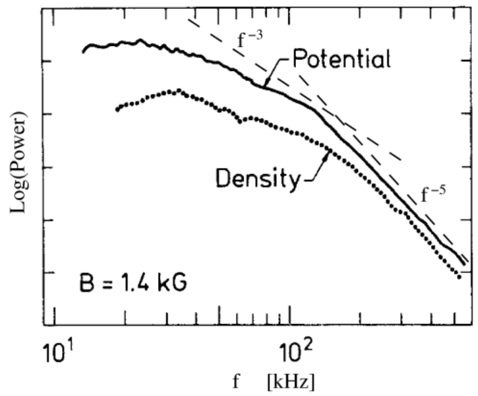

Fig. 2. Drift wave spectra obtained in the rotating plasma column (Caesium) of a Q-machine [7]. Power spectra for both density and electrostatic potential fluctuations are shown.

Most theoretical studies deal with wavenumber spectra, while it is frequency spectra that are most easily detected experimentally. In such cases it is usually argued with reference to Taylor's hypothesis (often called "the frozen turbulence hypothesis") that if the medium moving at velocity $\mathbf{U}$ is flowing sufficiently rapidly past an observer it is justified to associate the observed frequency with the Doppler shift, i.e. take $\omega \approx \mathbf{U}\cdot\mathbf{k}$. The experiment giving Fig. 2 made efforts to make this hypothesis valid, but often it is poorly argued. By this argument it was found that a frequency 500 kHz in Fig. 2 approximately corresponded to an average ion Larmor radius. The power spectra shown refer to scales larger than an average ion Larmor radius.

2. Observation in the Earth's ionosphere

Drift waves have been detected also in the Earth's ionosphere by instrumented rockets and satellites. Conditions in equatorial regions are at least qualitatively comparable to those found in laboratories, but in the Polar Regions the conditions are fundamentally different. Here the plasma density has a basic magnetic field aligned gradient in addition to possible gradients $\perp$ $\mathbf{B}_0$. To illustrate this effect we consider a simplified yet realizable model where we let the ions be cold, $T_i = 0$, and assume the electrons to be isothermally Boltzmann distributed at all times at temperature T_e.

First we ignore plasma density gradients $\perp$ $\mathbf{B}_0$. The electron pressure sets up a steady state electric field that balances the gravitational force on the ions, $\mathbf{E} = (Mg/e)\mathbf{1}_z$, where $\mathbf{1}_z$ is a unit vector along the $\mathbf{z}$-direction and M is the average ion mass. Assuming quasi-neutrality we find the plasma density $n(z) = n_0 \exp(-z/L_v)$ with $L_v = C_s^2/g$ with C_s being the ion sound speed. For a typical ion sound speed of 500 m s^{-1} found in the E-region we have $L_v = 25$ km.

Considering electrostatic waves propagating in the vertical direction we find the simple linear dispersion relation

$$\omega^2 = C_s^2 k_z^2 (1 + i\, M\, g/T_e\, k_z). \qquad (2)$$

This dispersion relation has solutions that increase in time for real k_z, or increase in the vertical direction for a real externally imposed frequency ω. A characteristic frequency is found to be $\Omega_{BV} = L_v/C_s = g/C_s$, which can be seen as a plasma equivalent of the Brunt-Väisälä frequency. At first sight such seemingly unstable solutions can appear unphysical since the system has no free energy by construction. The amplitude increase with increasing altitude is merely a consequence of the decreasing plasma density: for fluctuating plasma velocities u, the vertical component of the wave energy density flux is to lowest order $n(z)Mu^2C_s$. The ion sound velocity C_s is independent of plasma density, so to have a constant vertical energy flux, we must have u^2 increasing when n is decreasing and vice versa [10]. For large wavenumbers k_z it was found that linear ion

Landau damping dominated the growth obtained by (2) but for long wavelengths the phenomena should be observable. The results obtained by (2) have found experimental support, at least qualitatively [11].

We now allow for the plasma density to have a gradient also in the direction $\perp \mathbf{B}_0$ so that $n_0 = n_0(\mathbf{r}_\perp, z)$ with the subscript $\perp$ denoting a direction perpendicular to $\mathbf{B}_0 \| \mathbf{1}_z$. The drift waves can couple to the waves accounted for by (2) through the vertical wavenumber component k_z in (1). The analysis becomes a bit more complicated [4], but allows the analytical result for a linear wave dispersion relation

$$\omega^2(1 + k_\perp^2 a_i^2) - C_s\, \omega\, a_i\, \mathbf{k}_\perp \times \mathbf{1}_z \cdot \nabla_\perp \ln n_0(\mathbf{r}_\perp, z)$$

$$- i\, g\, k_z - C_s^2 k_z^2 = 0. \qquad (3)$$

Also this dispersion relation has solutions that can increase in space or time. The nature of this growth is the same as for (2). We have assumed Boltzmann distributed electrons, i.e. ignored collisions. In order to have turbulent conditions we need resistivity or free energy in terms of, for instance, electron beams [6]. Since space plasmas allow very small values of k_z we expect that large linear instability growth rates can be found since $\sigma_\|$ can become substantial. Indeed, relatively large amplitude electrostatic drift wave fluctuations have been observed.

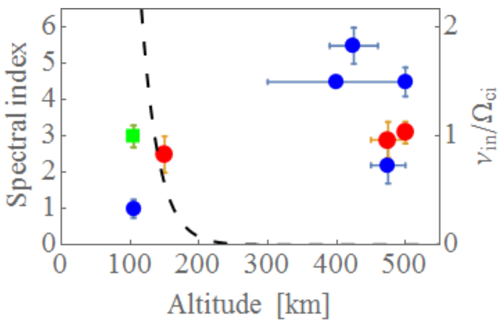

Fig. 3. Spectral indexes of drift wave turbulence as observed by instrumented rockets. The power spectral index variation with altitude for density is shown with blue circles, electric field in red and one measurement for potential in green. The dashed line (with scale to the right of the figure) shows the normalized altitude variation of the ion-neutral collision frequency $\nu_{in}(z)/\Omega_{ci}$.

Power spectra indicating strongly turbulent conditions have also been detected in the Earth's near and more distant ionospheres. In Fig. 3 we show a summary of some data reported in the literature. As long as the effects of neutral collisions can be ignored, i.e. for altitudes exceeding approximately 120 km, we find reasonable agreement with the theoretical results mentioned before [8], i.e. a k^{-5} variation for spectra of potential fluctuations, consistent with a k^{-3} power law for fluctuations of electric fields. As far as plasma density fluctuations are concerned we expect them to follow the potential at least for moderate amplitudes. One point at 475 km altitude in Fig. 3 falls significantly outside the others by giving approximately $k^{-2.5}$ for the density spectrum but it is plausible that this point should be interpreted as a "production" subrange and not a "coupling subrange" mentioned before.

For conditions where the ion neutral collisions dominate the observed power laws disagree with the analytical results by indicating a spectrum varying like k^{-1} for the fluctuations in plasma density. We argue that this limit has to be interpreted in terms of not the classical resistive drift wave instability [5], but by including the Hall or electro-jet current in the ionospheric E-region. When a steady state electric field directed $\perp \mathbf{B}_0$ is imposed on the ionosphere we can have the Farley-Buneman (FB) instability excited in addition to a gradient drift instability [12]. The criterion for this is that the relative ion and electron drift exceeds the ion sound speed, a condition that happens frequently. The FB-instability excites predominantly short wavelengths that will flatten the steep k^{-5} spectral variation. This hypothesis was tested in a laboratory experiment [13] with conditions like those illustrated in Fig. 1. By inlet of neutral gas the relative importance of ion-neutral collisions could be increased and the change in power-law relations for the turbulent spectra could be analysed. As argued before a flattening of the potential spectra were observed. It can also be noted that differences between density and potential spectra become larger for increasing neutral collision frequencies. For low neutral pressures the power spectra of density and potential fluctuations are nearly the same, at least in what is called the coupling subrange where fluctuation amplitudes are moderate. This can be expected for conditions where electrons can maintain a near Boltzmann equilibrium, i.e. for a small but non-vanishing plasma resistivity. As the neutral density increases, electron-neutral collisions inhibit the electron motion and for large neutral pressures we find the fluctuating density closer to being proportional to the fluctuating electric field.

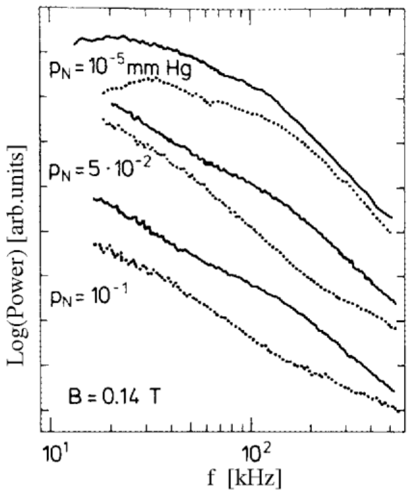

Fig. 4. Laboratory studies of the variation of spectra for varying neutral pressure [13] in the experiment shown schematically in Fig. 1. Full lines give potential spectra, dotted lines density spectra.

3. Conclusions

Some basic features of electrostatic drift waves have been summarized with particular attention to comparisons between laboratory and space observations. Conditions in laboratory and equatorial regions can be argued to be similar, but in the Polar Regions the basic linear models require some amendments, giving features particularly relevant for fluctuations propagating nearly perpendicular to the magnetic field. Space observations by rockets indicate that in the ionospheric E- and F-regions a consistent model can be obtained if the effects of neutral collisions are included also.

When data from instrumented satellites are included in the data base, the seemingly universal features of the turbulent drift wave spectra are in variance with some of the laboratory observations. The variations do not appear to be systematic and it is not evident that variation in plasma parameters can explain the differences in power spectra. Satellite data deserve further scrutiny.

4. References

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